



Math Placement Exam

Question 1 (5 points)

Solve for x . Be sure to check your work:

$$5x^2 + 10x = -5$$

Solution: $x = -1$

No Partial Credit

Question 2 (5 points) x^2-x-2

Solve for x . Be sure to check your work:

$$\frac{1}{2}\sqrt{4x+8} = x$$

Solution: $x = 2$

Partial Credit: 3 points for $x=-1$ and $x=2$ without checking $x=-1$

Question 3 (10 points)

$$Q = \frac{2}{p_1 p_2} + \frac{4p_1 - 5}{p_1 + p_2}$$

- (a) Find the derivative of Q with respect to p_1

Solution(5pts):

$$\frac{\partial Q}{\partial p_1} = -\frac{2}{p_1^2 p_2} + \frac{4p_2 + 5}{(p_1 + p_2)^2}$$

No Partial Credit.

- (b) Find the derivative of Q with respect to p_2

Solution(5pts):

$$\frac{\partial Q}{\partial p_2} = -\frac{2}{p_1 p_2^2} - \frac{4p_1 - 5}{(p_1 + p_2)^2}$$

No Partial Credit.

Question 4 (5 points)

Suppose x_1 is the number of hours you study for the GRE, x_2 is your parental income (measured in tens of thousands of dollars), and y is your GRE score in the following equation:

$$y = 45 + 2.2x_1 + 4x_2$$

- (a) Interpret the coefficients of the equation.

Solution (2pts, 1 for each coefficient):

- For every 1 extra hour studied, GRE score increases by 2.2 points on average, holding parental income constant.
- For every \$10,000 increase in parental income, GRE score increases by 4 points on average, holding constant the hours studied.
- (optional, interpretation of intercept): The predicted GRE score for someone who studied 0 hours and has a parental income of \$0 is 45 points.

- (b) How much does one's expected GRE score increase by when they study 5 more hours? What about when their parental income increases by \$5,000?

Solution (3 pts):

- 11 points
- 2 points

No Partial Credit

Question 5 (5 points)

True or False (with a brief 1-2 sentence explanation)

The median is more affected by outliers than the mean.

Solution: False; replacing the highest or lowest value in your dataset with an outlier will change the mean, but not the median.

Partial Credit: 2 points for correct answer, 3 points for correct explanation.

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Question 6 (10 points)

Suppose $U = x^2y^3$.

(a) Solve for the ratio of the first derivatives:

$$\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}$$

Solution(5pts):

$$\frac{2y}{3x}$$

No Partial Credit

(b) This ratio is called the “Marginal Rate of Substitution” (MRS). Does it increase or decrease as x increases?

Solution(5pts): Decreases

No Partial Credit

Question 7 (10 points)

Suppose $Q_D = -2p + 100$ represents the demand of a good, where p is the price of the good. Graph this equation with **p on the y-axis (vertical)** and Q_D on the x-axis (horizontal). No need to make the graph extremely pointwise precise, but just make the intercept and slope clearly visible.

Solution(5pts): (Graph of a standard line with a slope of $-\frac{1}{2}$ and intercept of 50); 5 points

Partial Credit: 2 points if student correctly graphs the relationship with Q on the vertical axis. 3 points if student graphs relationship with P on the vertical axis but has wrong slope or intercept.

If $Q_S = 2p + 40$ represents the quantity supplied, find the price where $Q_D = Q_S$ is true. This point of intersection is called the "Equilibrium". What is the equilibrium price **and** quantity?

Solution (5 pts):

$$p = 15, \quad Q = 70$$

Partial Credit: 3 points if student only reports equilibrium price or equilibrium quantity, or gets one of the two wrong.

Question 8 (10 points)

Suppose you have \$3000 available to spend on Macs and cheese. Macs cost \$500 each, and cheese costs \$100 each (hey, it's high quality!). Write down an expression that represents your total expenditure on Macs and Cheese.

Solution (5pts):

$$500M + 100C$$

No Partial Credit

If you spent the entire \$3,000, write down an equation that gives you the amount of cheese you can purchase if you purchase x units of Macs. Be sure to clearly define the variables you use. (Note: The slope of the equation above is called the "price ratio" and should equal the price of good x divided by the price of good y .)

Solution (5pts):

$$C = 30 - 5x$$

Partial Credit: Condition on the previous answer. If the previous answer was " $400M+200C$ " then one can receive full credit for the second part using " $C=30-4x$."

Question 9 (10 points)

Calculate Mean, Median, and Sample Variance of the following dataset:

$$\{6, 6, 12, 9, 7\}$$

Hint: The formula for Sample Variance is

$$\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2$$

where \bar{x} is the sample mean and N is the sample size

Solution:

Mean: 8 (2pts)

Median: 7(2pts)

Sample Variance: 6.5 or $\frac{26}{4}$ (6pts)

Question 10 (10 points)

Political scientists often are tasked with describing someone's political ideology using a single number. Suppose professor McMillon at the University of Chicago has developed a scale ranging from -5 to 5 , where -5 means extremely progressive and 5 means extremely conservative, and 0 is an exact moderate. Sarah Palin is considering entering the presidential race.

- (a) We know that Donald Trump's ideology is a 4.5 , and that Sarah Palin's ideology, S , is within 6 units of Trump's. Represent Sarah Palin's ideology using an **absolute value inequality**, and then solve this inequality to get a range for S .

Solution (3pts):

$$|S - 4.5| \leq 6$$

Which then simplifies to $-1.5 \leq S \leq 5$ or $[-1.5, 5]$

Partial Credit: 1 point if correctly answered without an absolute value inequality

- (b) We know that Joe Biden's ideology is a -2 , and that Sarah Palin's ideology is within 2 units of Biden's. Represent Sarah Palin's ideology using an **absolute value inequality**, and then solve this inequality to get a range for S .

Solution(3pts):

$$|S + 2| \leq 2$$

Which then simplifies to $-4 \leq S \leq 0$ or $[-4, 0]$

Partial Credit: 1 point if correctly answered without an absolute value inequality

- (c) If both (a) and (b) above are true, give an overall range of possible ideologies for Sarah Palin. (Note: If you think no such range exists, explain why).

Solution(4pts): Overlap of $[-1.5, 5]$ from (a) and $[-4, 0]$ from (b) gives:

$$-1.5 \leq S \leq 0$$

Partial Credit: Condition on the first two answers. One can receive full credit for part c even if parts a and b are wrong as long as there is consistency.

Question 11 (10 points)

Suppose $U = 2xy$. Subject to the constraint that $x + 2y = 40$, solve for the x and y values that maximize U .

Solution:

$$x = 20, \quad y = 10$$

Partial Credit: 3 points for using the constraint to make the correct substitution. 3 points for correctly taking derivatives and first order conditions. 2 points for each correct answer.

Question 12 (10 points)

The price of Charmin Ultra-soft Toilet Paper is given by the expression $P = 100 - y$, where y is the number of rolls sold. If the cost of producing y rolls of toilet paper is given by $C = y^2$, then what quantity y and price P would maximize the profits from selling toilet paper? Hint: The formula for profit is Revenue minus Cost, where Revenue is price (P) times quantity sold (y).

Solution:

$$y = 25, \quad P = 75$$

Partial Credit: 4 points for correctly writing out the profit in terms of y . 2 points for correctly writing the first order condition. 2 points for each correct answer.