Math Self-Assessment

Instructions

- You have 1.5 hours to complete the exam. Calculators are not allowed.
- Show your work in the space provided – there is partial credit! In particular, please write out formulas before you use them.
- Your answers should be simplified – e.g. take square roots with integer solutions, reduce fractions, etc. If a root or logarithm has a non-integer value, leave it in the precise form – e.g. do not attempt to write the approximate decimal values for ln 2 or 3.

Question 1 (5 points)

Solve for \( x \). Be sure to check your work:

\[ 5x^2 + 10x = -5 \]
Question 2 (5 points)

Solve for $x$. Be sure to check your work:

\[ \frac{1}{2} \sqrt{4x + 8} = x \]
Question 3 (10 points)

\[ Q = \frac{2}{p_1 p_2} + \frac{4p_1 - 5}{p_1 + p_2} \]

(a) Find the derivative of \( Q \) with respect to \( p_1 \)

(b) Find the derivative of \( Q \) with respect to \( p_2 \)
Question 4 (5 points)

Suppose $x_1$ is the number of hours you study for the GRE, $x_2$ is your parental income (measured in tens of thousands of dollars), and $y$ is your GRE score in the following equation:

$$y = 45 + 2.2x_1 + 4x_2$$

(a) Interpret the coefficients of the equation.

(b) How much does one's expected GRE score increase by when they study 5 more hours? What about when their parental income increases by $5,000$?
Question 5 (5 points)

True or False (with a brief 1-2 sentence explanation)

The median is more affected by outliers than the mean.
Question 6 (10 points)

Suppose $U = x^2 y^3$.

(a) Solve for the ratio of the first derivatives:

$$\frac{\partial U}{\partial x} \quad \frac{\partial U}{\partial y}$$

(b) This ratio is called the “Marginal Rate of Substitution” (MRS). Does it increase or decrease as $x$ increases?
Question 7 (10 points)

Suppose \( Q_D = -2p + 100 \) represents the demand of a good, where \( p \) is the price of the good. Graph this equation with \( p \) on the y-axis (vertical) and \( Q_D \) on the x-axis (horizontal). No need to make the graph extremely pointwise precise, but just make the intercept and slope clearly visible.

If \( Q_S = 2p + 40 \) represents the quantity supplied, find the price where \( Q_D = Q_S \) is true. This point of intersection is called the “Equilibrium”. What is the equilibrium price and quantity?
Question 8 (10 points)

Suppose you have $3000 available to spend on Macs and cheese. Macs cost $500 each, and cheese costs $100 each (hey, it’s high quality!). Write down an expression that represents your total expenditure on Macs and Cheese.

If you spent the entire $3,000, write down an equation that gives you the amount of cheese you can purchase if you purchase \( x \) units of Macs. Be sure to clearly define the variables you use. (Note: The slope of the equation above is called the “price ratio” and should equal the price of good \( x \) divided by the price of good \( y \).)
Question 9 (10 points)

Calculate Mean, Median, and Sample Variance of the following dataset:

\{6, 6, 12, 9, 7\}

Hint: The formula for Sample Variance is

\[
\frac{1}{N - 1} \sum_{i=1}^{N} (x_i - \bar{x})^2
\]

where \(\bar{x}\) is the sample mean and \(N\) is the sample size
Question 10 (10 points)

Political scientists often are tasked with describing someone’s political ideology using a single number. Suppose professor McMillon at the University of Chicago has developed a scale ranging from $-5$ to 5, where $-5$ means extremely progressive and 5 means extremely conservative, and 0 is an exact moderate. Sarah Palin is considering entering the presidential race.

(a) We know that Donald Trump’s ideology is a 4.5, and that Sarah Palin’s ideology, $S$, is within 6 units of Trump’s. Represent Sarah Palin’s ideology using an absolute value inequality, and then solve this inequality to get a range for $S$.

(b) We know that Joe Biden’s ideology is a $-2$, and that Sarah Palin’s ideology is within 2 units of Biden’s. Represent Sarah Palin’s ideology using an absolute value inequality, and then solve this inequality to get a range for $S$.

(c) If both (a) and (b) above are true, give an overall range of possible ideologies for Sarah Palin. (Note: If you think no such range exists, explain why).
Question 11 (10 points)

Suppose $U = 2xy$. Subject to the constraint that $x + 2y = 40$, solve for the $x$ and $y$ values that maximize $U$. 
Question 12 (10 points)

The price of Charmin Ultra-soft Toilet Paper is given by the expression $P = 100 - y$, where $y$ is the number of rolls sold. If the cost of producing $y$ rolls of toilet paper is given by $C = y^2$, then what quantity $y$ and price $P$ would maximize the profits from selling toilet paper? Hint: The formula for profit is Revenue minus Cost, where Revenue is price ($P$) times quantity sold ($y$).