

# MATHEMATICAL METHODS FOR PhD AND MACRM

Summer 2018

---

<b>Instructor:</b>	Liqun Liu	<b>Email:</b> <i>liqunliu@uchicago.edu</i>
<b>Teaching assistant:</b>	TBD	TBD
<b>Time:</b>	MTWRF 9:00 – 12:00	<b>Place:</b> Room 289A

---

**Course Pages:** [TBD](#)

**Office Hours:** By appointment.

**Recitation Session:** 1:00-2:00 pm

**Registration:** Harris PhD and MACRM students.

**Overview:** In modern days, one cannot understand economic theory without a working knowledge of real and convex analysis. This course aims to prepare students to succeed in the first year Harris PhD core courses, and provide a solid foundation for future technical trainings.

The tentative plan is as follows: we kick off with a rigorous treatment of mathematical analysis on metric space, followed by a detour into linear algebra and differential equation. We conclude multivariate analysis by the end of week two. In week three, we discuss convex analysis, optimization, and monotone comparative statics. Time permitting, we will also cover basic measure-theoretical probability. Motivating examples are drawn from economic theory.

Undoubtedly, the course covers a lot. I do not expect you to grasp all the details at your first try. Instead, I would like to convey to you the big-picture ideas that facilitate self-studies in your subsequent coursework. As for me, I find it beneficial to ask myself the following questions after class: why should we care about this mathematical concept? Where should we place the concept vis-a-vis others? How should we apply the concepts to economic problems?

I can hardly think of a way to understand mathematics other than (relentless) practice. To this end, I will assign daily problem set. Problem sets will be discussed in recitation session every afternoon. To incentivize your efforts, I will assign everyday quizzes drawn from homework at the beginning of lectures.

**Prerequisites:** Two-semester multivariate calculus and one semester linear algebra, plus certain “mathematical maturity”.

## Main References:

- Cinlar, Erhan and Robert Vanderbei (CV), *Real and Convex Analysis*, Springer, 2012.
- Hirsch, Morris, Stephen Smale and Robert Devaney (HSD), *Differential Equations, Dynamic Systems, and An Introduction to Chaos*. Elsevier, 3rd ed. 2013
- Jehle, Geoffrey and Philip Reny (JR), *Advanced Microeconomic Theory*, Addison Wesley Longman, 2nd ed. 2001
- Ok, Efe, *Real Analysis with Economic Application*, Princeton, 2007
- Vohra, Rakesh, *Advanced Mathematical Economics*, Routledge 2005

## Optional Texts

- Aliprantis, Charalambos and Kim Border, *Infinite Dimensional Analysis: A Hitchhikers Guide*, New York, Springer-Verlag, 1999.
- Corbae, Dean, Maxwell Stinchcombe and Juraj Zeman, (DSZ) *An Introduction to Mathematical Analysis for Economic Theory and Econometrics*, Princeton, 2009

## Main Topics

### 1. Metric Space (5 Lectures)

- Basics: set theory, binary relation, equivalence class
- Metric space: real line, topology, convergence, completeness, compactness, Heine-Borel, connectedness, intermediate value theorem, continuity

Main reference: CV

### 2. Linear Algebra, Differential Calculus and ODE (5 Lectures)

- Linear algebra: vector space, linear dependence, matrix, determinants, eigenvalues and eigenvectors, diagonalization,
- First and second order ODE
- Normed vector space, derivative, mean value theorem, implicit/inverse function theorem

Main reference: Vohra, HSD

### 3. Convex Analysis and Optimization (2 Lectures)

- Convex Analysis: convex sets, separating hyperplanes, (quasi) concave functions
- Optimization: unconstrained/constrained optimization, Karush-Kuhn-Tucker condition

Main reference: Vohra, DSZ, JR

### 4. Advanced Topics (3 Lectures)

- Fixed point theorem: Banach, Brouwer, Kakutani
- Lattice theory and supermodularity, monotone comparative statics
- Envelope Theorem

Main reference: Ok, Vohra, Yildiz's lecture notes,