THE UNIVERSITY OF CHICAGO
Harris Public Policy

Harris Mathematics Pre-Core Program
Curriculum Overview

The Harris Mathematics Pre-Core Program is a three-week review course designed to ensure that incoming Master's students are proficient in algebra and calculus. The course prepares students for the mandatory math exam, which is held during orientation week. One core faculty member leads the course with the support of several teaching assistants. The course meets every weekday and includes a review of algebra principles from 8:30 to 9:30 a.m., a review of calculus principles from 10:00 a.m. to 12:00 p.m., and TA sessions and office hours in the afternoon. The following reviews both the algebra and calculus topics covered in the course.

Algebra Topics

Basics of Algebra & Geometry
Topics:
- Rules of algebra
- Linear equations
- Coefficients and variables
- Graphing
- Area and volume

Example Assessment:
- Simplify expressions using the rules of algebra.
- Solve equations of one variable, and solve equations of multiple variables for one of the variables.
- Perform algebraic manipulations on equations with coefficients expressed as parameters rather than as specific values.
- Graph equations of two variables.
- Calculate and interpret the slope of a line.
- Calculate distances between points.
- Express the equation of a line in multiple forms.
- Find the intercepts for any line.
- Identify parallel and perpendicular lines based on their slopes.
- Given two points, determine the line through the points.

Resources:
Manipulating expressions: https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-manipulating-expressions/v/combining-like-terms

Linear equations: https://www.khanacademy.org/math/algebra/two-var-linear-equations-and-intro-to- functions/standard-form

Distance & midpoint: https://www.khanacademy.org/math/geometry-home/analytic-geometry-topic#distance-and-midpoints

Parallel & perpendicular lines: https://www.khanacademy.org/math/geometry-home/analytic-geometry-topic#parallel-and-perpendicular

Rational expressions: https://www.khanacademy.org/math/algebra2/rational-expressions

Exponents: https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/world-of- exponents-college-readiness/v/introduction-to-exponents

Order of operations: https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/algebra-foundations-order-of-operations/v/introduction-to-order-of-operations

Radicals (roots): https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/square-roots-for-college/v/understanding-square-roots

Fractions: https://www.khanacademy.org/math/arithmetic/fraction-arithmetic

Systems of Equations
Topics:
- Solving by substitution
- Solving by addition (elimination)
- Systems of 2 and 3 variables
- Graphical interpretation of systems

Example Assessment:
- Solve particular systems, including nonlinear equations.
- Graph linear systems.
- Interpret graphs of systems.
- Convert a graphical system into an algebraic system

Resources:
Solving by substitution: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems- topic/cc-8th-systems-with-substitution

Solving by elimination: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems- topic/cc-8th-systems-elimination

Graphical systems: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems-topic/cc-8th-systems-graphically

Inconsistent systems: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems-topic/cc-8th- systems-solutions/v/inconsistent-systems-of-equations

Infinite solutions: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems-topic/cc-8th-systems-solutions/v/infinite-solutions-to-systems

Three variables: https://www.khanacademy.org/math/algebra2/advanced-equations-and-inequalities/systems-with-three-variables/v/systems-of-three-variables

Non-linear systems: https://www.khanacademy.org/math/algebra/quadratics/systems-of-quadratic-equations/v/non-linear-systems-of-equations-1

Exponents & Logarithms

Topics:
- Rules of algebra with exponents & logarithms
- Expressing roots as exponents / fractional exponents
- Negative exponents / expressing division using exponents
- Exponents with base e
- Logarithms as the inverse of exponents
- Using logarithms to solve problems with variables in the exponent
- Logarithms of different bases
- The natural log, ln

Example Assessment:
- Simplify expressions with logarithms
- Solve equations and inequalities with variables in the exponent
- Solve equations with a variable inside a logarithm
- Use the rules of algebra with logarithms to expand or contract the terms inside a logarithm

Resources:
Exponents: https://www.khanacademy.org/math/algebra-home/alg-exp-and-log#alg-exp-prop-review

Logarithms: https://www.khanacademy.org/math/algebra2/logarithms-tutorial

Exponential functions: https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/graphs-of-basic-exponential-functions/v/graphing-exponential-functions

Growth and decay: https://www.khanacademy.org/math/algebra2/exponential-growth-and-decay-alg-2

Logarithmic functions: https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions#logarithmic-equations

Summation Notation
Topics:
- Using summation notation to express series and sequences
- Formulae for calculating infinite sums
- Simplifying complex sums
- Calculating sums

Example Assessment:
- Compute a numerical sum or expand an abstract sum (finite or infinite)
- Simplify a complex expression in summation notation
- Express a sequence of terms using summation notation
- Translate a scenario described in words to an expression in summation notation

Resources:
Sigma notation: https://www.khanacademy.org/math/integral-calculus/sequences-series-approx-calc/calculus-series/v/sigma-notation-sum

Summation notation: https://www.khanacademy.org/math/integral-calculus/sequences-series-approx-calc/calculus-series/v/writing-series-sigma-notation

Inequalities & Absolute Values
Topics:
- Rules of algebra for inequalities & absolute values
- Solving inequalities
- Solving equations and inequalities with absolute values
- Graphing sets (used to express the solution to an inequality)

Example assessment:
- Solve and graph the solution to equations and inequalities, including those with absolute value (where the variable may be inside the absolute value, outside the absolute value, or both)
- Represent the solutions to inequalities and absolute value problems graphically

Resources:
Solving inequalities: https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities/alg1-one-step-inequalities/v/inequalities-using-multiplication-and-division

Solving absolute value: https://www.khanacademy.org/math/algebra/absolute-value-equations-functions/absolute-value-equations/v/absolute-value-equations

Inequalities with absolute value: https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities/compound-inequalities/v/compound-inequalities

Graphing linear inequalities: https://www.khanacademy.org/math/algebra-basics/core-algebra-graphing-lines-slope/core-algebra-graphing-linear-inequalities/v/graphing-inequalities

Quadratic inequalities: https://www.khanacademy.org/math/algebra2/advanced-equations-and-inequalities/quadratic-inequalities/v/quadratic-inequality-example

Systems of inequalities: https://www.khanacademy.org/math/algebra/two-variable-linear-inequalities/graphing-inequalities/v/graphical-system-of-inequalities

Graphing absolute values: https://www.khanacademy.org/math/algebra/absolute-value-equations-functions/graphs-of-absolute-value-functions/e/graphs-of-absolute-value-functions

Functions
Topics:
- The nature of a function
- Domain and range
- Common functional forms, including polynomial, radical, exponential, logarithmic and rational
- Composite functions
- Inverse functions

Example Assessment:
- Evaluate a function at a point
- Find the - and - intercepts of the graphs of a function
- Find the domain or range of a function
- Find a composite
- Break a composite function into constituent functions
- Based on the graph of a function, quickly determine how the graph changes as the function is shifted and/or reflected (sum, difference, product, quotient)
- Find the inverse of a function where one exists

Resources:

Functions: https://www.khanacademy.org/math/algebra/algebra-functions/intro-to-functions/v/what-is-a-function

Domain and range: https://www.khanacademy.org/math/algebra2/advanced-functions/determining-the-range-of-a-function/v/domain-and-range-of-a-function-given-a-formula

Composing Functions: https://www.khanacademy.org/math/algebra2/manipulating-functions/funciton-composition/v/function-composition

Inverse functions: https://www.khanacademy.org/math/algebra2/manipulating-functions/introduction-to-inverses-of-functions/v/introduction-to-function-inverses

Shifting and reflecting: https://www.khanacademy.org/math/algebra2/manipulating-functions/stretching-functions/v/shifting-and-reflecting-functions

Polynomials

Topics:
- Factoring polynomials
- Distributing factored polynomials
- Quadratic formula
- Solving / roots of polynomials
- Binomial theorem

Example assessment:
- Find and interpret the roots of a polynomial
- Find intercepts of factored polynomials
- Factor a polynomial
- Identify the degree of the polynomial
- Use the above approaches to solve inequalities and absolute value problems
- Identify the maximum number of roots of a generic polynomial
- Use the binomial theorem to expand $(\ +\ )^n$ or determine a particular coefficient of the expansion

Resources:
Quadratic factoring: https://www.khanacademy.org/math/algebra/polynomial-factorization#factoring-polynomials-2-quadratic-forms

Quadratic formula: https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/using-the-quadratic-formula

Binomial theorem: https://www.khanacademy.org/math/algebra2/polynomial-functions/binomial-theorem/v/binomial-theorem

Calculus Topics

I. SINGLE VARIABLE CALCULUS

Functions, Limits & Continuity
Topics:
- Functions include absolute value functions, rational functions, polynomials, exponential, logarithmic functions (the laws of logs).
- Domain and Range of Functions.
- How to Compute Limits.
- Definition of Continuity.

Example Assessment:
- Determine the domain of different types of functions.
- Compute limits: includes the different algebra tricks needed to take limits.

Resources:
Domain functions: https://www.khanacademy.org/math/algebra2/advanced-functions/domain-of-advanced-functions/e/domain_of_a_function

Limit and make: https://www.khanacademy.org/math/differential-calculus/limits-topic/continuity-limits/v/fancy-algebra-to-find-a-limit-and-make-a-function-continuous

Derivatives
Topics:
- Rates**,** Slopes and the Definition of the Derivative
- Definition of a derivative and Differentiability.
- Derivative Theorems (includes Product Rule, Chain Rule).
- First Derivative Test; Second Derivative Test
- Global and local minimums and maximums (1 variable, local & global min max).
- Analysis of Functions: Second derivatives, Concavity and Inflection Points.

Example Assessment:
- Compute a derivative using the definition of limits.
- Take a derivative of different functions without using the definition of a limit (power rule).
- Use the product rule, quotient rule, chain rule when taking derivatives.
- Compute second derivatives of a variety of functions.
- Identify critical points of a function.
- Identify local and global minima and maxima.
- Identify inflection points.
- Determine whether functions are concave up or concave down and over what intervals.
- Make an accurate graph of a function.

Resources:
Derivatives as slope: https://www.khanacademy.org/math/differential-calculus/taking-derivatives/derivative-intro/v/calculus-derivatives-1-new-hd-version

Derivative intuition: https://www.khanacademy.org/quetzalcoatl/deprecated-exercises/deprecated-bucket-5/v/derivative-intuition-module

Concavity: https://www.khanacademy.org/math/differential-calculus/derivative-applications/concavity-inflection-points/v/concavity-concave-upwards-and-concave-downwards-intervals

Integration - Area and the Definite Integral

Topics:

- Definition of Integration.
- How to compute integrals.
- Anti-derivatives and the Fundamental Theorem of Calculus.

Example Assessment

- Computing integrals of relatively easy functions.

Resources:

Simple definite integral: https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/definite-integrals/v/evaluating-simple-definite-integral

II. **MULTIVARIABLE CALCULUS**

Functions of Several Variables

Topics:

- Explanation of functions of several variables.
- Examples.

Levels Sets of Functions

Topics:

- Explanation of level sets of functions.
- Examples.

Resources:

Level sets: http://mathinsight.org/level_sets

Partial Derivatives

Topics:

- Partial Derivatives.
- Implicit differentiation.

Example Assessment:

- Compute partial derivatives and the gradient.
- Finding the derivative using implicit differentiation.

Resources:

Partial derivatives: https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives#partial-derivatives

Implicit differentiation: https://www.khanacademy.org/math/ap-calculus-ab/advanced-differentiation-ab/implicit-differentiation-intro-ab/v/implicit-differentiation-1

Gradients & Lagrange Multiplier

Topics:
- Computing the gradient.
- Lagrange Multipliers.
- Constrained Optimization using the Lagrange multiplier.

Example Assessment:
- Solve a problem of constrained optimization using the Lagrange multiplier.

Resources:

Multivariable calculus: http://freevideolectures.com/Course/2300/Multivariable-Calculus/13