Mathematical Methods for PhD and MACRM

Summer 2017

Instructor:	Liqun Liu	**Time:**	MTWRF 9:00 – 12:00
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Course Pages: TBD

Office Hours: TBD

Registration: This course is primarily designed to review and develop necessary mathematical tools for Harris PhD and MACRM students.

Overview: In modern days, one cannot understand the development of economic theory without a working knowledge of real and convex analysis. This course serves to prepare students to succeed in the first year Harris PhD core courses, and provide a solid foundation for future technical trainings.

The tentative plan is as follows: we kick off by a rigorous treatment of mathematical analysis on metric space, followed by a detour into linear algebra and basic ordinary differential equation. Upon finishing analysis by the end of week two, we start our journey in convex analysis and optimization with motivating examples from economic theory. Finally, we introduce modern techniques widely used in deriving comparative statics, namely monotone comparative statics.

Undoubtedly, the course covers a lot. I do not expect you to grasp all the details at your first try. Nonetheless, it is extremely helpful to have a big picture in mind after lectures, and thereby develop a feel of mathematics that facilitates self-study in your subsequent coursework. In particular, I find it beneficial to ask myself the following questions: why should we care about this mathematical concept? Where should we place the concept vis-a-vis others? How should we apply the concepts to economic problems?

I can hardly think of a way to understand mathematics other than (relentless) practice. To this end, I will assign daily problem set. Problem sets will be discussed in recitation session every afternoon. I will not collect problem sets; instead I will assign a quiz with one question drawn from the previous problem set at the beginning of every class. Your performance is evaluated by quizzes and the final exam.

Prerequisites: Two-semester multivariate calculus and one semester linear algebra. No prior knowledge is required beyond that. Of course, certain mathematical maturity is always a plus.

Main References:

- Boller, John and Paul Sally, *Advanced Calculus*, Unpublished manuscript
- Cinlar, Erhan and Robert Vanderbei (CV), *Real and Convex Analysis*, Springer, 2012.
- Jehle, Geoffrey and Philip Reny (JR), *Advanced Microeconomic Theory*, Addison Wesley Longman, 2nd ed. 2001
- Kreps, David, *Microeconomic Foundations I*, Princeton, 2013.
- Mas-Colell, Andrew, Michael Whinston, and Jerry Green (MWG), *Microeconomic Theory*, Oxford, 1995.

- Ok, Efe, *Real Analysis with Economic Application*, Princeton, 2007
- Vohra, Rakesh, *Advanced Mathematical Economics*, Routledge 2005

Main Topics

1. Metric Space (5 Lectures)

   - Basics: set theory, binary relation, equivalent class
   - Metric space: real line, topology, convergence, completeness, compactness, Heine-Borel, connectedness, Intermediate Value Theorem, continuity

   Main reference: CV

2. Linear Algebra, Multivariate Analysis and Differential Equation (5 Lectures)

   - Applied: matrix, determinants, eigenvalues and eigenvectors, diagonalization,
   - Theory: (normed) vector space, linear dependence
   - Derivative as linear transformation, directional derivative, total derivative, fundamental theorem of calculus
   - First and second order ODE

   Main reference: Shannon's lecture notes and Vohra's book for the theoretical part; MWG's appendix for application

3. Convex Analysis and Optimization (3 Lectures)

   - Convex Analysis: concave functions, convex sets, convex hull, extreme points, Caratheodory Theorem, separating hyperplanes
   - Optimization: unconstrained, constrained optimization, Karush-Kuhn-Tucker condition

   Main reference: CV, Kreps, MWG, and JR

4. Advanced Topics (2 Lectures)

   - Fixed point theorem: Banach, Brouwer, Kakutani
   - Lattice theory and supermodularity, monotone comparative statics
   - Envelope Theorem

   Main reference: Vohra's book, Yildiz's lecture notes, Ashworth and Bueno de Mesquita (2006), Milgrom and Segal (2002)