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POLITICAL ECONOMY OF REDISTRIBUTION

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POLITICAL ECONOMY OF REDISTRIBUTION

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It is often argued that additional constraints on redistribution such as granting veto power to more players in society better protects property from expropriation. We use a model of multilateral bargaining to demonstrate that this intuition may be flawed. Increasing the number of veto players or raising the supermajority requirement for redistribution may reduce protection on the equilibrium path. The reason is the existence of two distinct mechanisms of property protection. One is formal constraints that allow individuals or groups to block any redistribution that is not in their favor. The other occurs in equilibrium where players without such powers protect each other from redistribution. Players without formal veto power anticipate that the expropriation of other similar players will ultimately hurt them and thus combine their influence to prevent redistributions. In a stable allocation, the society exhibits a "class" structure with class members having equal wealth and strategically protecting each other from redistribution.

KEYWORDS: Political economy, legislative bargaining, property rights, institutions.

1. INTRODUCTION

ECONOMISTS HAVE LONG VIEWED PROTECTION OF PROPERTY RIGHTS as a cornerstone of efficiency and economic development (e.g., Coase (1937), Alchian (1965), Hart and Moore (1990)). Yet, from a political economy perspective, property rights should be understood as equilibrium outcomes rather than exogenous constraints. Legislators or, more generally, any political actors cannot commit to entitlements, prerogatives, and rights. Whether property rights are effectively protected depends on the political economy of the respective society and its institutions. The idea that granting veto power to different actors in the society enhances protection dates back at least to the Roman republic (Polybius [2010], Machiavelli 1515[1984]) and, in modern times, to Montesquieu's *Spirit of the Laws* (1748[1989]) and the Federalist papers, the intellectual foundation of the United States Constitution. In essay number 51, James Madison argued for the need to contrive the government "as that its several constituent parts may, by their mutual relations, be the means of keeping each other in their proper places." Riker (1987) concurs: "For those who believe, with Madison, that freedom depends on countering ambition with ambition, this constancy of federal conflict is a fundamental protection of freedom."

In modern political economy, an increased number of veto players has been associated with beneficial consequences. North and Weingast (1989) argued that the British parliament, empowered at the expense of the crown by the Glorious Revolution in 1688, provided "the credible commitment by the government to honour its financial agreement [that] was part of a larger commitment to secure private rights." Root (1989) demonstrated that this allowed British monarchs to have lower borrowing costs compared to the French kings. In Persson, Roland, and Tabellini (2000), separation of taxing and spending decisions within budgetary decision-makings improves the accountability of elected officials and limits rent-seeking by politicians.

We study political mechanisms that ensure protection against expropriation by a majority. In practice, institutions come in different forms such as the separation of powers

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between the legislative, executive, and judicial branches of government, multicameralism, federalism, supermajority requirements, and other constitutional arrangements that effectively provide some players with veto power. One of the first examples was described by Plutarch [2010]: the Spartan *Gerousia*, the Council of Elders, could veto motions passed by the *Apella*, the citizens' assembly. In other polities, it might be just individuals with guns who have effective veto power. Essentially, all these institutions allow individuals or collective actors to block any redistribution without their consent. If we interpret property rights as institutions that allow holders to prevent reallocations without their consent, then we can formally investigate the effect of veto power on the allocation of property.

In addition to property rights, formalized in constitutions or codes of law, that is, game forms in a theoretical model, property rights might be protected as equilibrium outcomes of interaction of strategic economic agents. The property rights of an individual may be respected not because he is powerful enough to protect them on his own, that is, has veto power, but because others find it in their respective interest to protect his rights. Specifically, members of a coalition, formed in equilibrium, have an incentive to oppose the expropriation of each other because they know that once a member of the group is expropriated, others will be expropriated as well. As a result, the current allocation of assets might be secure even in the absence of explicit veto power.

If property rights may emerge from strategic behavior of rational economic agents, such rights are necessarily dynamic in nature. A status quo allocation of assets stays in place for the next period, unless it is changed by the political decision mechanism in which case the newly chosen allocation becomes the status quo for the next period. This makes models of legislative bargaining with the endogenous status quo (following Baron (1996) and Kalandrakis (2004) the natural foundation for studying political economy of redistribution and protection of property from expropriation.[2]

In our model, agents, some of whom have veto power, decide on allocation of a finite number of units. If the (super)majority decides on redistribution, the new allocation becomes the status quo for the next period. We start by showing that non-veto players build coalitions to protect each other against redistribution. Diermeier and Fong (2011) demonstrated that with a sole agenda-setter, two other players could form a coalition to protect each other from expropriation by the agenda-setter. However, this feature is much more general: our Propositions 1–3 show that such coalitions form in a general multilateral setting with any number of veto players. The size of a protective coalition is a function of the supermajority requirement and the number of veto players. Example 1 demonstrates that with five players, one of whom has veto power, three non-veto players with equal wealth form a coalition to protect each other.

EXAMPLE 1: Consider five players who decide how to split ten indivisible units of wealth, with the status quo being $(1, 2, 3, 4; 0)$. Player #5 is the sole veto player and proposer, any reallocation requires a majority of votes, and we assume that when players are indifferent, they support the proposer. In a standard legislative bargaining model, the game ends when a proposal is accepted. Then player #5 would simply build a coalition to expropriate two players, say #3 and #4, and capture the surplus resulting in $(1, 2, 0, 0; 7)$. However, this logic does not hold in a dynamic model where the agreed

[2]Recent contributions to this literature include Anesi and Seidmann (2014, 2015), Anesi and Duggan (2015, 2017), Baron and Bowen (2015), Bowen and Zahran (2012), Diermeier and Fong (2011, 2012), Duggan and Kalandrakis (2012), Kalandrakis (2010), Richter (2014), Vartiainen (2014), and Nunnari (2016). We discuss the existing literature and its relationship to our results in Section 5.

upon allocation can be redistributed in the subsequent periods. That is, with the new status quo $(1, 2, 0, 0; 7)$, player #5 would propose to expropriate players #1 and #2 by moving to $(0, 0, 0, 0; 10)$, which is accepted in equilibrium. Anticipating this, players #1 and #2 should not agree to the first expropriation, thus becoming the effective guarantors of property rights of players #3 and #4. Starting with $(1, 2, 3, 4; 0)$, the ultimate equilibrium allocation might be either $(3, 3, 3, 0; 1)$ or $(2, 2, 2, 0; 4)$ or $(2, 2, 0, 2; 4)$; in any of the cases, at least three players will not be worse off. In general, an allocation is stable if and only if there is a group of three non-veto players of equal wealth, and the remaining non-veto player has an allocation of zero.

The fact that all non-veto players who are not expropriated in Example 1 have the same wealth in the ultimate stable allocation is not accidental. With a single proposer, we cannot isolate the impact of veto power from the impact of agenda-setting power; non-veto players have no chance to be agenda-setters, and their action space is very limited. With multiple veto players and multiple agenda-setters without veto power, we demonstrate that the endogenous veto groups have a certain "class structure": in a stable allocation, most of the non-veto players are subdivided into groups of equal size, within each of which individual players have the same amount of wealth, whereas the rest of the society is fully expropriated. While we make specific assumptions to single out equilibria to focus on, the "class structure" is robust (see Section 5 for the discussion).

EXAMPLE 2: Consider the economy as in Example 1, yet four votes, rather than three, are required to change the status quo. Now, if the initial status quo is $(1, 2, 3, 4; 0)$, which is unstable, the ultimate stable allocation will be $(1, 3, 3, 1; 2)$, that is, two endogenous veto groups will be formed (players #1 and #4 form one, and #2 and #3 form the other). In general, with five players—one veto player and four votes required to change the status quo—all stable sets are of the form, up to permutations, $(x_1, x_2, x_3, x_4; x_5)$ with $x_1 = x_2$ and $x_3 = x_4$. This is the simplest example of a society exhibiting a nontrivial class structure.

The number and size of these endogenous classes vary as a function of the number of veto players and the supermajority requirements. Perhaps paradoxically, adding additional exogenous protection (e.g., by increasing the number of veto players) may lead to the breakdown of an equilibrium with stable property rights, as the newly empowered player (the one who was granted or has acquired veto power) now no longer has an incentive to protect the others. Thus, by adding additional hurdles to expropriation in the form of veto players or supermajority requirements (see Example 4 below), the protection of property rights may in fact be eroded. In other words, players' property may be well protected in the absence of formal constraints, while strengthening formal constraints may result in expropriation. Our next example demonstrates this effect more formally.

EXAMPLE 3: As in Example 1, there are five players and three votes are required to make a change, but now there are two veto players instead of one, #4 and #5. Allocations $(x_1, x_2, x_3; x_4, x_5)$, in which at least one of players #1, #2, or #3 has zero wealth and at least one has a positive amount, are unstable, as the two veto players will obtain the vote of one player an allocation of zero and redistribute the assets of the remaining two players. One can prove that an allocation is stable if and only if $x_1 = x_2 = x_3$ (up to a permutation). This means that if we start with $(3, 3, 3, 0; 1)$, which was stable with one veto player, making player #1 an additional veto player will destroy stability. As a result, the society will move either to an allocation in which all 10 units are split between the two veto players or to some allocation where the non-veto players form an endogenous veto group that protects its members from further expropriation, for example, $(4; 1, 1, 1; 3)$, in which #2–4 form such group.

We see here an interesting phenomenon. The naive intuition would suggest that giving one extra player (player #1 in this example) veto power would make it more difficult for player #5 to expropriate the rest of the group. However, the introduction of a new veto player breaks the stable coalition of non-veto players and makes #5 more powerful. Before the change, non-veto players sustained an equal allocation, precisely because they were more vulnerable individually. With only one veto player and an equal allocation for players #1, #2, and #3, the three non-veto players form an endogenous veto group, which blocks any transition that hurts the group as a whole (or even one of them). An additional veto player makes expropriation more, not less, likely. Note that both the amount of wealth being redistributed and the number of players affected by expropriation are significant. The number of players who stand to lose is two, close to half of the total number of players, and at least 4 units, close to half of the total wealth, is redistributed through voting. In Proposition 4, we show that the class structure, which is a function of the number of veto players and the supermajority requirement, determines a limit to the amount of wealth redistributed after an exogenous shock to one player's wealth.

In addition to granting veto rights, changes to the decision-making rule (e.g., the degree of supermajority) can also have a profound, yet somewhat unexpected effect on protection of property. Higher supermajority rules are usually considered safeguards that make expropriation more difficult, as one would need to build a larger coalition. The next example shows that this intuition is flawed as well: in a dynamic environment, increasing the supermajority requirements may trigger additional redistribution.

EXAMPLE 4: As above, there are five players who make redistributive decisions by majority, and one of whom (#5) has veto power. Allocation $(3, 3, 3, 0; 1)$ is stable. Now, instead of a change in the number of veto players, consider a change in the supermajority requirements. If a new rule requires four votes, rather than three, the status quo allocation becomes unstable. Instead, a transition to one of the allocations that become stable, $(3, 3, 0, 0; 4)$ or $(4, 4, 0, 0; 2)$, will be supported by coalition of four players out of five. (The veto player, #5, benefits from the move, #4 is indifferent as he gets 0 in both allocations, and #1 and 2 will support this move as they realize that with the new supermajority requirement they form a group that is sufficient to protect its members against any expropriation.) Thus, an increase in supermajority may result in expropriation and redistribution.

As Example 4 demonstrates, raising the supermajority requirement does not necessarily strengthen property rights, as some players are expropriated as a result. Proposition 6 establishes that this phenomenon, as well as the one discussed in Example 3, is generic: adding a veto player or raising the supermajority requirement almost always leads to a wave of redistribution. To obtain the comparative statics results described in Examples 3 and 4 (Propositions 5 and 6), we use a general characterization of politically stable allocations in a multilateral-negotiations settings (Proposition 3). These results contrast with the existing consensus in the literature, summarized by Tsebelis (2002): "As the number of veto players of a political system increase, policy stability increases."

Redistribution through overtaxation (e.g., Persson and Tabellini (2000)) or an outright expropriation (e.g., Acemoglu and Robinson (2006)) has been the focus of political economy studies since at least Machiavelli (1515)[1984] and Hobbes (1651)[1991]. A large number of works explored the relationship between a strong executive and his multiple subjects (e.g., Greif (2006) on the institute of *podesteria* in medieval Italian cities; Haber, Razo, and Maurer (2003) on the 19th century Mexican presidents; or Guriev and Sonin (2009) on Russian oligarchs). Acemoglu, Robinson, and Verdier (2004) and Padro i Miquel (2007) build formal divide-and-rule theories of expropriation, in each of which a powerful executive exploited the existing cleavages for personal gain. In addition to the multilateral bargaining literature, policy evolution with endogenous quo is studied, among others, in Dixit, Grossman, and Gul (2000), Hassler, Storesletten, Mora, and Zilibotti (2003), Dekel, Jackson, and Wolinsky (2009), Battaglini and Coate (2007, 2008), and Battaglini and Palfrey (2012). To this diverse literature, our model adds the emergence of "class politics"; also, we demonstrate that introduction of formal institutions of property rights protection might result, in equilibrium, in less protection than before.

The remainder of the paper is organized as follows. Section 2 introduces our general model. In Section 3, we establish the existence of (pure-strategy Markov perfect) equilibrium in a non-cooperative game and provide full characterization of stable wealth allocations. Section 4 focuses on the impact of changes in the number of veto players or supermajority requirements. In Section 5, we discuss our modeling assumptions and robustness of our results, while Section 6 concludes. The Supplemental Material (Diermeier, Egorov, and Sonin (2017)) contains technical proofs and some additional examples and counterexamples.

2. SETUP

Consider a set N of $n = |N|$ political agents who allocate a set of indivisible identical objects between themselves. In the beginning, there are b objects, and the set of feasible allocations is therefore

$$\mathbf{A} = \left\{ x \in (\mathbb{N} \cup \{0\})^n : \sum_{i=1}^{n} x_i \le b \right\}.$$

We use lower index x_i to denote the amount player i gets in allocation $x \in \mathbf{A}$ throughout the paper, and we denote the total number of objects in allocation x by $\|x\| = \sum_{i \in N} x_i$.

Time is discrete and indexed by $t > 0$, and the players have a common discount factor β. In each period t, the society inherits x^{t-1} from the previous period (x^0 is given exogenously) and determines x^t through an agenda-setting and voting procedure. A transition from x^{t-1} to some alternative $y \in \mathbf{A}$ is feasible if $\|y\| \le \|x^{t-1}\|$; in other words, we allow for the objects to be wasted, but not for the creation of new objects.[3] For a feasible alternative y to defeat the status quo x^{t-1} and become x^t, it needs to gain the support of a sufficiently large coalition of agents.

To define which coalitions are powerful enough to redistribute, we use the language of winning coalitions. Let $V \subset N$ be a nonempty set of veto players (denote $v = |V|$; without loss of generality, let us assume that V corresponds to the last v agents $n - v + 1, \dots, n$), and let $k \in [v, n]$ be a positive integer. A coalition X is *winning* if and only if (a) $V \subset X$ and (b) $|X| \ge k$. The set of winning coalitions is denoted by

$$\mathcal{W} = \{X \in 2^N \setminus \{\emptyset\} : V \subset X \text{ and } |X| \ge k\}.$$

[3]An earlier version of the model required that there is no waste, so $\|x^t\| = \|x^0\| = b$ throughout the game, and the results were identical. In principle, the possibility of waste can alter the set of outcomes in a legislative bargaining model (e.g., Richter (2014)).

In this case, we say that the society is governed by a k rule with veto players V, meaning that a transition is successful if it is supported by at least k players and no veto player opposes it. We will compare the results for different k and v. We maintain the assumption that there is at least one veto player—that V is nonempty—throughout the paper; this helps us capture various political institutions such as a supreme court. We do not require that $k > n/2$, so we allow for minority rules. For example, a 1-rule with the set of veto players $\{i\}$ is a dictatorship of player i.

Our goal is to focus on redistribution from politically weak players to politically powerful ones, and especially on the limits to such redistribution. We thus introduce the following assumption to enable veto players to buy the votes of those who would otherwise be indifferent. In each period, there is an arbitrarily small budget that the players can distribute in this period; its default size is ε, and there is another ε for each unit transferred from non-veto players to veto players. Furthermore, to avoid equilibria where non-veto players shuffle the units between themselves, we assume that there is a small transition cost $\delta \in (0, \varepsilon)$ that is subtracted from the budget every time there is a transition.[4] A feasible proposal in period t is therefore a pair (y, ξ) such that $y \in \mathbf{A}$ that satisfies $\|y\| \leq \|x^{t-1}\|$ and $\xi_i \in \mathbb{R}^n$ satisfies $\xi_i \geq 0$ for all $i \in N$ and $\|\xi_i\| \leq (1+\max(\sum_{i\in V} y_i - \sum_{i\in V} x_i^{t-1}, 0)) \times \varepsilon - \mathbf{I}\{y \neq x^{t-1}\} \times \delta$. Throughout the paper, we assume $0 < \delta < \varepsilon < \frac{1-\beta}{b+1}$. (We will show that as $\varepsilon, \delta \to 0$, the equilibria converge to some equilibria of the game where $\varepsilon = \delta = 0$; thus, focusing on equilibria that may be approximated in this way may be thought of as equilibrium refinement that rules out uninteresting equilibria, specifically the ones that feature cycles.[5])

The timing of the game below uses the notion of a protocol, which might be any finite sequence of players (possibly with repetition); for existence results, however, we require it to end with a veto player.[6] We denote the set of protocols by

$$\Pi = \bigcup_{\eta=1}^{\infty} \{\pi \in N^{\eta} : \pi_{\eta} \in V\}.$$

The protocol to be used is realized in the beginning of each period, taken from a distribution $\mathcal{D}$ that has full support on Π (to save on notation, we assume that each veto player is equally likely to be the last one, but this assumption does not affect our results). If the players fail to reach an agreement, the status quo prevails in the next period. Thus, in each period t, each agent i gets instantaneous utility $u_i^t = x_i^t + \xi_i^t$ and acts so as to maximize his continuation utility

$$U_i^t = u_i^t + \mathbb{E} \sum_{j=1}^{\infty} \beta^j u_i^{t+j},$$

where the expectation is taken over the realizations of the protocols in the subsequent periods. We focus on the case where the players are sufficiently forward looking; specifi-

[4]In most models of multilateral bargaining, it is standard to assume that whenever an agent is indifferent, she agrees to the proposal (see Section 5). Otherwise, the proposer would offer an arbitrarily small amount to an indifferent player. In our model, we assume indivisible units, but allow for such infinitesimal transfers.

[5]The working paper version Diermeier, Egorov, and Sonin (2013) contains a variant of such a game with corresponding refinements.

[6]Allowing non-veto players to propose last may in some cases lead to nonexistence of protocol-free equilibria as Example A2 in the Supplementary Material (Diermeier, Egorov, and Sonin (2017)) demonstrates.

cally, we assume $\beta > 1 - \frac{1}{b+2}$.[7] More precisely, the timing of the game in period $t \geq 1$ is as follows.

Stage 1. Protocol π^t is drawn from the set of possible protocols Π.

Stage 2. For $j = 1$, player π_j^t is recognized as an agenda-setter and proposes a feasible pair (z^j, χ^j), or passes.

Stage 3. If player π_j^t passes, the game proceeds to Stage 5; otherwise, all players vote, sequentially, in the order given by protocol π^t, *yes* or *no*.

Stage 4. If the set of those who voted *yes*, Y^j, is a winning coalition, that is, if $Y^j \in \mathcal{W}$, then the new allocation is $x^t = z^j$, the transfers are $\xi^t = \chi^j$, and the game proceeds to Stage 6. Otherwise, the game proceeds to the next stage.

Stage 5. If $j < |\pi^t|$, where $|\pi|$ denotes the length of protocol π, then the game moves to Stage 2 with j increased by 1. Otherwise, the society keeps the status allocation $x^t = x^{t-1}$, and the game proceeds to the next stage.

Stage 6. Each player i receives an instantaneous payoff u_i^t.

The equilibrium concept we use is Markov perfect equilibrium (MPE). In any such equilibrium σ, the transition mapping $\phi = \phi^\sigma : \mathbf{A} \times \Pi \to \mathbf{A}$, which maps the previous period's allocation and the protocol realization for the current period into the current period's allocation, is well defined. In what follows, we focus on protocol-free equilibria (protocol-free MPE[8]), namely, σ such that $\phi^\sigma(x, \pi) = \phi^\sigma(x, \pi')$ for all $x \in \mathbf{A}$ and $\pi, \pi' \in \Pi$. We thus abuse notation and write $\phi = \phi^\sigma : \mathbf{A} \to \mathbf{A}$ to denote the transition mapping of such equilibria.

3. ANALYSIS

Our strategy is as follows. We start by proving some basic results about equilibria of the non-cooperative game described above. Then we characterize stable allocations, that is, allocations with no redistribution, and demonstrate that the stable allocations correspond to equilibria of the non-cooperative game. We then proceed to study comparative statics with respect to the number of veto players, supermajority requirements, and equilibrium paths that follow an exogenous shock to some players' wealth.

3.1. *Non-Cooperative Characterization*

Consider a protocol-free MPE σ, and let $\phi = \phi_\sigma$ be the transition mapping that is generated by σ and defined at the end of Section 2. (Using transition mappings, rather than individuals' agenda-setting and voting strategies, allows us to capture equilibrium paths in terms of allocations and transitions, i.e., in a more concise way). Iterating the mapping ϕ gives a sequence of mappings $\phi, \phi^2, \phi^3, \ldots : \mathbf{A} \to \mathbf{A}$, which must converge if ϕ is acyclic. (Mapping ϕ is *acyclic* if $x \neq \phi(x)$ implies $x \neq \phi^\tau(x)$ for any $\tau > 1$; we will show that every MPE satisfies this property.) Denote this limit by ϕ^∞, which is simply ϕ^τ for some τ as the set $\mathbf{A}$ is finite. We say that mapping ϕ is *one-step* if $\phi = \phi^\infty$ (this is equivalent to $\phi = \phi^2$), and we call an MPE σ *simple* if ϕ_σ is one-step. Given an MPE σ, we call allocation x *stable* if $\phi_\sigma(x) = x$. Naturally, ϕ_σ^∞ maps any allocation into a stable allocation.

Our first result deals with the existence of an equilibrium and its basic properties.

[7]This condition means that a player prefers $x + 1$ units tomorrow to x units today, for any $x \leq b + 1$. This assumption is relatively weak compared to models of multilateral bargaining that require β to approach 1.

[8]See Examples A4 and A5 in the Supplemental Material, where allowing for non-Markov strategies or dropping the requirement that transitions be the same for every protocol can lead to counterintuitive equilibria.

PROPOSITION 1: *Suppose* $\beta > 1 - \frac{1}{b+2}$, $\varepsilon < \frac{1-\beta}{b}$, *and* $\delta < \frac{\varepsilon}{n}$. *Then the following statements hold*:

(i) *There exists a protocol-free Markov perfect equilibrium* σ.
(ii) *Every protocol-free MPE is acyclic*.
(iii) *Every protocol-free MPE is simple*.
(iv) *Every protocol-free MPE is efficient in that it involves no waste* (*for any* $x \in \mathbf{A}$, $\|\phi(x)\| = \|x\|$).

These results are quite strong, and are made possible by the requirement that the equilibrium be protocol-free. For a fixed protocol, equilibria might involve multiple iterations before reaching a stable allocations (see Example A3 in the Supplemental Material). However, these other equilibria critically depend on the protocol and are therefore fragile; in contrast, transition mappings supported by protocol-free MPE are robust (e.g., they would remain if the protocols are taken from a different distribution, for example).

The proof of Proposition 1 is technically cumbersome and is relegated to the Supplemental Material. However, the idea is quite straightforward. We construct a candidate transition mapping ϕ_σ that we want to be implemented in the equilibrium. If the society starts the period in state $x = x^{t-1}$ such that $\phi(x) = x$, we verify that it is a best response for the veto players to block any transitions except for those that are blocked by a coalition of non-veto players and, thus, x remains intact. If the society starts the period in state x such that $\phi(x) \neq x$, we verify that there is a feasible vector of small transfers that may be redistributed from those who strictly benefit from such transition to those who are indifferent, and that the society would be able to agree on such a vector over the course of the protocol. The second result, the acyclicity of MPE, relies on the presence of transaction costs, which rules out the possibility of non-veto players shuffling units among themselves (Example A1 in the Supplementary Material exhibits cyclic equilibria that would exist in the absence of this assumption). To show that every protocol-free MPE is simple, we show that if there were an allocation from which the society would expect to reach a stable allocation in exactly two steps, then for a suitable protocol it would instead decide to skip the intermediate step and transit to the stable allocation immediately. Finally, given that every MPE is simple, the society may always allocate the objects that would otherwise be wasted to some veto player (e.g., the proposer) without facing adverse dynamic consequences ("the slippery slope"), which ensures that each transitions involves no waste and the allocations are efficient.

The following corollary highlights that the possibility of small transfers may be viewed as an equilibrium refinement.

COROLLARY 1: *Suppose that for game* Γ *with parameter values* β, ε, δ *as in Proposition* 1, $\phi = \phi_\sigma$ *is the transition mapping that corresponds to a protocol-free MPE* σ. *Then consider game* Γ' *with the same* $\beta' = \beta$, *but* $\varepsilon' = \delta' = 0$. *Then there exists protocol-free MPE* σ' *with the same transition mapping* $\phi_{\sigma'} = \phi$.

The equilibrium transitions described in Proposition 1 are not necessarily unique as the following Example 5 demonstrates. Still, an allocation stable in one of such equilibria is stable in all such equilibria.

EXAMPLE 5: Suppose there are $b = 3$ units of wealth, four agents, the required number of votes is $k = 3$, and the set of veto players is $V = \{\#4\}$. In this case, there is a simple equilibrium with transition mapping ϕ, under which allocations

$(0,0,0;3)$, $(1,1,0;1)$, $(1,0,1;1)$, and $(0,1,1;1)$ are stable. Specifically, we have the transitions $\phi(2,1,0;0)=\phi(1,2,0;0)=(1,1,0;1)$, $\phi(0,2,1;0)=\phi(0,1,2;0)=(0,1,1;1)$, $\phi(2,0,1;0)=\phi(1,0,2;0)=\phi(1,1,1;0)=(1,0,1;1)$, and any allocation with $x_4=2$ has $\phi(x)=(0,0,0;3)$. However, another mapping ϕ' coinciding with ϕ except that $\phi'(1,1,1;0)=(1,1,0;1)$ may also be supported in equilibrium.

3.2. *Stable Allocations*

Our next goal is to get a more precise characterization of equilibrium mappings and stable allocations. Let us define a dominance relation $\rhd$ on $\mathbf{A}$ as

$$y \rhd x \iff \|y\| \leq \|x\| \quad \text{and} \quad \{i \in N : y_i \geq x_i\} \in \mathcal{W} \quad \text{and} \quad y_j > x_j \qquad \text{for some } j \in V.$$

Intuitively, allocation y dominates allocation x if transition from x to y is feasible and some powerful player prefers y to x strictly so as to be willing to make this motion, and also there is a winning coalition that (weakly) prefers x to y. Note that this does not imply that y will be proposed or supported in an actual voting against x because of further changes this move may lead to. Following the classic definition von Neumann and Morgenstern (1947), we call a set of states $\mathbf{S} \subset \mathbf{A}$ *von Neumann–Morgenstern-stable* (vNM-stable) if the following two conditions hold: (i) For no two states $x, y \in \mathbf{S}$ it holds that $y \rhd x$ (*internal stability*) and (ii) for each $x \notin \mathbf{S}$ there exists $y \in \mathbf{S}$ such that $y \rhd x$ (*external stability*).

The role of this dominance relation for our redistributive game is demonstrated by the following result.

PROPOSITION 2: *For any protocol-free MPE σ, the set of stable allocations $\mathbf{S}_\sigma = \{x \in \mathbf{A} : \phi_\sigma(x) = x\}$ is a von Neumann–Morgenstern-stable set for the dominance relation $\rhd$.*

Proposition 2 implies that the fixed points of transition mappings of non-cooperative equilibria described in Proposition 1 correspond to a von Neumann–Morgenstern-stable set. Our next result states that such a stable set is also unique; this implies, in particular, that for any two protocol-free MPE σ and σ', the set of stable allocations is identical. Consequently, we are able to study stable allocations irrespective of a particular equilibrium of the bargaining game.

The following Proposition 3 gives a precise characterization of stable allocations. To formulate it, let us denote $m = n - v$, the number of non-veto players; $q = k - v$, the number of non-veto players that is required in any winning coalition; $d = m - q + 1 = n - k + 1$, the size of a minimal blocking coalition of non-veto players; and, finally, $r = \lfloor m/d \rfloor$, the maximum number of pairwise disjoint blocking coalitions that non-veto players may be split into.

PROPOSITION 3: *For the binary relation $\rhd$, a vNM-stable set exists and is unique.*[9] *Each element x of this set $\mathbf{S}$ has the following structure: the set of non-veto players $M = N \setminus V$ may be split into a disjoint union of r groups $G_1, \dots, G_r$ of size d and one (perhaps empty) group G_0 of size $m - rd$, such that inside each group, the distribution of wealth is equal:*

[9]Proposition A1 in the Supplemental Material proves this set is also the largest consistent set (Chwe (1994)).

$x_i = x_j = x_{G_k}$ whenever $i, j \in G_k$ for some $k \geq 1$, and $x_i = 0$ for any $i \in G_0$. In other words, $x \in \mathbf{S}$ if and only if the non-veto players can be permuted in such a way that

$$x = (\underbrace{\lambda_1, \dots, \lambda_1}_{d\ times}, \underbrace{\lambda_2, \dots, \lambda_2}_{d\ times}, \dots, \underbrace{\lambda_r, \dots, \lambda_r}_{d\ times}, \underbrace{0, \dots, 0}_{m-rd\ times}; \underbrace{x_{m+1}, \dots, x_n}_{veto\ players})$$

for some $\lambda_1 \geq \cdots \geq \lambda_r \geq 0$ such that $d \sum_{j=1}^{r} \lambda_j + \sum_{l=1}^{n-m} x_{m+l} \leq b$.

The proof of this result is important for understanding the structure of endogenous veto groups, and we provide it in the text. We show that starting from any wealth allocation $x \in \mathbf{S}$, it is impossible to redistribute the units between non-veto players without making at least d players worse off, and thus no redistribution would gain support from a winning coalition. In contrast, starting from any allocation $x \notin \mathbf{S}$, such redistribution is possible. Furthermore, our proof will show that there is an equilibrium where in any transition, the set of individuals who are worse off is limited to the $d-1$ richest non-veto players.

PROOF OF PROPOSITION 3: We will prove that set $\mathbf{S}$ is vNM-stable, thus ensuring existence. To show internal stability, suppose that $x, y \in \mathbf{S}$ and $y \rhd x$, and let the r groups be $G_1, \dots, G_r$ and $H_1, \dots, H_r$, respectively. Without loss of generality, we can assume that each set of groups is ordered so that x_{G_j} and y_{H_j} are nonincreasing in j for $1 \leq j \leq r$. Let us prove, by induction, that $x_{G_j} \leq y_{H_j}$ for all j.

The induction base is as follows. Suppose that the statement is false and $x_{G_1} > y_{H_1}$; then $x_{G_1} > y_s$ for all $s \in M$. This yields that for all agents $i \in G_1$, we have $x_i > y_i$. Since the total number of agents in G_1 is d, G_1 is a blocking coalition and, therefore, it cannot be true that $y_j \geq x_j$ for a winning coalition, contradicting that $y \rhd x$.

For the induction step, suppose that $x_{G_l} \leq y_{H_l}$ for $1 \leq l < j$, and also assume, to obtain a contradiction, that $x_{G_j} > y_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > y_{H_s}$. Consequently, for agent $i \in \bigcup_{l=1}^{j} G_l$ to have $y_i \geq x_i$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)d = d$ agents in $\bigcup_{l=1}^{j} G_l \subset M$, it cannot be the case that $y_i \geq x_i$, which contradicts the assumption that $y \rhd x$. This establishes that $x_{G_j} \leq y_{H_j}$ for all j and, therefore, $\sum_{i \in M} x_i \leq \sum_{i \in M} y_i$. But $y \rhd x$ would require that $x_i \leq y_i$ for all $i \in V$ with at least one inequality strict, which implies $\sum_{i \in N} x_i < \sum_{i \in N} y_i$, a contradiction to $\|y\| \leq \|x\|$. This proves internal stability of set $\mathbf{S}$.

Let us now show that the external stability condition holds. To do this, we take any $x \notin \mathbf{S}$ and will show that there is $y \in \mathbf{S}$ such that $y \rhd x$. Without loss of generality, we can assume that x_i is nonincreasing for $1 \leq i \leq m$ (i.e., non-veto players are ordered from richest to poorest). Let us denote $G_j = \{(j-1)d + 1, \dots, jd\}$ for $1 \leq j \leq r$ and $G_0 = M \setminus (\bigcup_{j=1}^{r} G_j)$. Since $x \notin \mathbf{S}$, it must be that either for some G_j, $1 \leq j \leq r$, the agents in G_j do not get the same allocation or they do, but some individual $i \in G_0$ has $x_i > 0$. In the latter case, we define y by

$$y_i = \begin{cases} x_i & \text{if } i \leq dr \text{ or } i > m+1, \\ 0 & \text{if } dr < i \leq m, \\ x_i + \sum_{j \in G_0} x_j & \text{if } i = m+1. \end{cases}$$

(In other words, we take everything possessed by individuals in G_0 and distribute it among veto players, for example, by giving everything to one of them.) Obviously, $y \in \mathbf{S}$ and $y \rhd x$.

If there exists a group G_j such that not all of its members have the same amount of wealth, let j be the smallest such number. For $i \in G_l$ with $l < j$, we let $y_i = x_i$. Take the first $d-1$ members of group G_j, $Z = \{(j-1)d+1, \dots, jd-1\}$. Together, they possess $z = \sum_{i=(j-1)d+1}^{jd-1} x_i > (d-1)x_{jd}$ (the inequality is strict precisely because not all x_i in G_j are equal). Let us now take these z units and redistribute them among all the agents (perhaps including those in Z) in the following way. For each $s : j < s < r$, we let $y_{(s-1)d} = y_{(s-1)d+1} = \cdots = y_{sd-1} = x_{(s-1)d}$. As the agent with number $(s-1)d$ was the richest among these d agents, they are weakly better off now that they have the same amount of wealth.

Now observe that in each group s, we spent at most $(d-1)(x_{(s-1)d} - x_{sd-1}) \leq (d-1)(x_{(s-1)d} - x_{sd})$. For $s = r$, we take d agents as $D = \{(r-1)d, \dots, m\} \cup Z'$, where $Z' \subset Z$ is a subset of the first $d - (m-(r-1)d+1) = rd - m - 1$ agents needed to make D a collection of exactly d agents (notice that $Z' = \emptyset$ if $|G_0| = d-1$ and $Z' = Z$ if $G_0 = \emptyset$). For all $i \in D$, we let $y_i = x_{(r-1)d}$ (making all members of G_0 weakly better off and spending at most $(d-1)x_{(r-1)d}$ units) and we let $y_i = 0$ for each $i \in Z \setminus Z'$. We have thus defined y_i for all $i \in M$ and distributed

$$c \leq (d-1)(x_{jd} - x_{(j+1)d} + \cdots + x_{(r-2)d} - x_{(r-1)d} + x_{(r-1)d}) = (d-1)x_{jd},$$

having $z - c > 0$ remaining at our disposal. As before, we let $y_{m+1} = x_{m+1} + z - c$ and $y_i = x_i$ for $i > m+1$. We have constructed $y \in \mathbf{S}$ such that $\|y\| = \|x\|$, $y_{m+1} > x_{m+1}$, and $\{i \in N : y_i < x_i\} \subset Z$. The latter, given $|Z| \leq d-1$, implies $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$, which means $y \rhd x$. This completes the proof of external stability, and thus $\mathbf{S}$ is vNM-stable.

Let us now show that $\mathbf{S}$ is a unique stable set defined by $\rhd$.[10] Suppose not, so there is $\mathbf{S}'$ that is also vNM-stable. Let us prove that $x \in \mathbf{S} \Leftrightarrow x \in \mathbf{S}'$ by induction on $\sum_{i \in M} x_i$. The induction base is trivial: if $x_i = 0$ for all $i \in M$, then $x \in \mathbf{S}$ by definition of $\mathbf{S}$. If $x \notin \mathbf{S}'$, then there must be some y such that $y \rhd x$. But for such y,

$$\sum_{i \in N} y_i \geq \sum_{i \in V} y_i > \sum_{i \in V} x_i = \sum_{i \in N} x_i,$$

which contradicts $\|y\| \leq \|x\|$.

The induction step is as follows. Suppose that for some x with $\sum_{i \in M} x_i = j > 0$, $x \in \mathbf{S}$ but $x \notin \mathbf{S}'$ (the reverse case is treated similarly). By external stability of $\mathbf{S}'$, $x \notin \mathbf{S}'$ implies that for some $y \in \mathbf{S}'$, $y \rhd x$, which in turn yields that $\sum_{i \in V} y_i > \sum_{i \in V} x_i$ and $\|y\| \leq \|x\|$. We have

$$\sum_{i \in M} y_i = \|y\| - \sum_{i \in V} y_i < \|x\| - \sum_{i \in V} x_i = \sum_{i \in M} x_i = j.$$

For y such that $\sum_{i \in M} y_i < j$ induction yields that $y \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}'$, and thus $y \in \mathbf{S}$. Consequently, there exists some $y \in \mathbf{S}$ such that $y \rhd x$, but this contradicts $x \in \mathbf{S}$. This contradiction establishes uniqueness of the stable set. *Q.E.D.*

Proposition 3 enables us to study the set of stable allocations $\mathbf{S}$ without reference to a particular equilibrium σ. The characterization obtained in this proposition gives several important insights. First, the set of stable allocations (fixed points of any transition mapping under any equilibrium) does not depend on the mapping; it maps into itself when

[10] An alternative (nonconstructive) way to prove uniqueness is to use a theorem by von Neumann and Morgenstern (1947) that states that if a dominance relation allows for no finite or infinite cycles, the stable set is unique.

either the veto players V or the non-veto players $N \setminus V$ are reshuffled in any way. Second, the allocation of wealth among veto players does not have any effect on stability of allocations. Third, each stable allocation has a well defined "class" structure: every non-veto player with a positive allocation is part of a group of size d (or a multiple of d) of equally endowed individuals who have incentives to protect each other's interests.[11]

To demonstrate how such protection works, consider the following example.

EXAMPLE 6: There are $b = 12$ units, $n = 5$ individuals with one veto player (#5), and a supermajority of four is needed for a transition ($k = 4$). By Proposition 3, stable allocations have two groups of size two. Let ϕ be a transition mapping for some simple MPE σ, and let us start with stable allocation $x = (4, 4, 2, 2; 0)$. Suppose that we exogenously remove a unit from player #2 and give it to the veto player; that is, consider $y = (4, 3, 2, 2; 1)$. Allocation y is unstable, and player #1 will necessarily be expropriated. However, the way redistribution may take place is not unique; for example, $\phi(y) = (3, 3, 2, 2; 2)$ is possible, but so is $\phi(y) = (2, 3, 3, 2; 2)$ or $\phi(y) = (2, 3, 2, 3; 2)$. Now suppose that one of the players possessing two units, say player #3, was expropriated, that is, take $z = (4, 4, 1, 2; 1)$. Then it is possible that the other member, player #4, would be expropriated as well: $\phi(z) = (4, 4, 1, 1; 2)$. But it is also possible that one of the richer players may be expropriated instead: for example, a transition to $\phi(z) = (4, 1, 1, 4; 2)$ would be supported by all players except #2.

Example 6 demonstrates that equilibrium protection that agents provide to each other may extend beyond members of the same group. In the latter case, player #2 would oppose a move from $(4, 4, 2, 2; 0)$ to $(4, 4, 1, 2; 1)$ if in the subgame the next move is to $(4, 1, 1, 4; 2)$. Thus, richer players might protect poorer ones, but not vice versa; as Proposition 4 below shows, this is a general phenomenon.

We see that, in general, an exogenous shock may lead to expropriation, on the subsequent equilibrium path, of players belonging to different wealth groups; the particular path depends on the equilibrium mapping, which is not unique. However, if we apply the refinement that only equilibria with a "minimal" (in terms of the number of units that need to be transferred) redistribution along the equilibrium path are allowed, then only the players with exactly the same wealth would suffer from the redistribution that follows a shock. More importantly, Example 6 demonstrates the mechanism of mutual protection among players with the same wealth. If a non-veto player becomes poorer, at least $d - 1$ other players would suffer in the subsequent redistribution. This makes them willing to oppose any redistribution from any of their members. Their number, if we include the initial expropriation target himself, is d, which is sufficient to block a transition. Thus, members of the same group have an incentive to act as a politically cohesive coalition, in which its members mutually protect each others' economic interests.

Proposition 3 also allows for the following simple corollary.

COROLLARY 2: *Suppose that in game Γ defined above, the set of stable allocations (in any protocol-free MPE) is* $\mathbf{S}$. *Take any integer $h > 1$, and consider the set of allocations $\mathbf{A}^h$ given by*

$$\mathbf{A}^h = \left\{x \in \left(\mathbb{R}^+\right)^n : \|x\| \leq b \text{ and } \forall i \in N, hx_i \in \mathbb{Z}\right\}.$$

[11]It is permissible that two groups have equal allocations, $x_{G_j} = x_{G_k}$, or that members of some or all groups get zero. In particular, any allocation x where $x_i = 0$ for all $i \in M$ is in $\mathbf{S}$. Notice that if non-veto players get the same under two allocations x and y, so $x|_M = y|_M$, then $x \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}$; moreover, this is true if $x_i = y_{\pi(i)}$ for all $i \in M$ and some permutation π on M.

Take $\beta_h > 1 - \frac{1}{bh+2}$, $\varepsilon_h < \frac{1-\beta}{b(h+1)}$, *and* $\delta_h < \varepsilon_h$. *Then the set of stable allocations* $\mathbf{S}^h$ *in the new game* Γ^h (*again, in any protocol-free MPE*) *satisfies* $\mathbf{S} \subset \mathbf{S}^h$.

In other words, taking a finer partition of units of redistributions (splitting each unit into h indivisible parts) preserves stable allocations. This result follows immediately from Proposition 3. It effectively says that even though our results are obtained under the assumption of a discrete number of indivisible units, they have a broader appeal: once dividing units into several parts is allowed, the stable allocations remain stable. This implies that the set $\mathbf{S}$ not only describes stable outcomes for any appropriately refined equilibrium within the game, but is also a robust predictor of stable allocations if the minimal units are redefined, provided, of course, that players interact frequently enough.[12]

The next proposition generalizes Example 6 so that one can better understand the mechanics of mutual protection. It highlights that protection of a non-veto player is sustained, in equilibrium, by equally endowed or richer individuals, rather than by those who have less wealth. Proposition 4 is formulated as follows. We take some equilibrium characterized in Proposition 3 and consider a stable allocation. Then we consider another, perturbed, allocation, in which one non-veto player has less wealth. We show that the resulting allocation is unstable, and compare the ultimate stable allocation with the initial, unperturbed one.

PROPOSITION 4: *Consider any MPE* σ *and let* $\phi = \phi_\sigma$. *Suppose that the voting rule is not unanimity* ($k < n$), *so* $d > 1$. *Take any stable allocation* $x \in \mathbf{S}$ *and some non-veto player* $i \in M$, *and let new allocation* $y \in \mathbf{A}$ *be such that* $y|_{M\setminus\{i\}} = x|_{M\setminus\{i\}}$ *and* $y_i < x_i$. *Then the following statements hold*:

(i) *Player i will never be as well off as before the shock, but he will not get any worse off*: $y_i \le [\phi(y)]_i < x_i$. *Furthermore, the number of players who suffer as a result of a redistribution on the equilibrium path defined by* σ *is given by*

$$\left|\left\{j \in M \setminus \{i\} : \left[\phi(y)\right]_j < y_j\right\}\right| = d - 1.$$

(ii) *Suppose, in addition, that for any* $k \in M$ *with* $x_k < x_i$, $x_k \le y_i$, *that is, the shock did not make player i poorer than the players in the next wealth group. Then* $[\phi(y)]_j < y_j$ *implies* $x_j \ge x_i$, *that is, members of poorer wealth groups do not suffer from redistribution.*

The essence of Proposition 4 is that following a negative (exogenous) shock to some player's wealth ($y_i < x_i$), at least $d - 1$ other players are expropriated, and player i never fully recovers. If the shock is relatively minor so the ranking of player i with respect to other wealth groups did not change (weak inequalities are preserved),[13] then it must be

[12]Notice that since the sequence of stable sets satisfies $\mathbf{S} \subset \mathbf{S}^2 \subset \mathbf{S}^3 \subset \cdots$, their limit is a well defined set $\mathbf{S}^\infty = \overline{\bigcup_{j>1} \mathbf{S}^j}$, where the overbar denotes topological closure. This set has the simple structure

$$\mathbf{S}^\infty = \left\{x \in \Delta \mid \exists \rho \in S^n : x_{\rho(1)} = \cdots = x_{\rho(d)}, x_{\rho(d+1)} = \cdots = x_{\rho(2d)}, \ldots, x_{\rho((r-1)d+1)} = \cdots = x_{\rho(rd)}\right\},$$

where Δ is the $(N-1)$-dimensional unit simplex and $\rho \in S^n$ is any permutation. However, for these limit allocations to be approached in the non-cooperative game that we study, one would have to take a sequence of discount factors β_j that tends to 1, so interactions should be more and more frequent. Intuitively, to study fine partitions of the state space, one would need finer partition of time intervals as well to prevent "undercutting." If this condition does not hold, veto players would be able to expropriate everything in the long run (see, e.g., Nunnari (2016)).

[13]Note that this will always be the case if, for example, $y_i = x_i - 1$.

equally endowed or richer people who suffer from subsequent redistribution. Thus, in the initial stable allocation x, they have incentives to protect i from the negative shock. This result may be extended to the case when a negative shock affects more than one (but less than d) non-veto players. The proof is straightforward when all the affected players belong to the same wealth group. However, this requirement is not necessary. If expropriated players belong to different groups, then the lower bound of the resulting wealth after redistribution is the amount of wealth that the poorest (post-shock) player possesses. In this case, the number of players who suffer as a result of the redistribution following the shock is still limited by $d-1$.

Our next step is to derive comparative statics with respect to different voting rules given by k and v.

4. COMPARING VOTING RULES

Suppose that we vary the supermajority requirement, k, and the number of veto players, v. The following result easily follows from the characterization in Proposition 3.

PROPOSITION 5: *Fix the number of individuals* n.

(i) *The size of each group* G_j, $j \geq 1$, *is decreasing as the supermajority requirement* k *increases. In particular, for* $k = v + 1$, $d = n - v = m$, *and thus all the non-veto players form a single group; for* $k = n$ (*unanimity rule*), $d = 1$, *and so each player can veto any change.*

(ii) *The number of groups is weakly increasing in* k, *from* 1 *when* $k = v + 1$ *to* m *when* $k = n$ (*from* 0 *when* $k < v + 1$).

(iii) *The size of each group* G_j, $j \geq 1$, *does not depend on the number of veto players, but as* v *increases, the number of groups weakly decreases, reaching zero for* $v > n - d$.

This result implies that the size of groups does not depend on the number of veto players, but only on the supermajority requirement as it determines the minimal size of blocking coalitions. As the supermajority requirement increases, groups become smaller. This has a very simple intuition: as redistribution becomes harder (it is necessary to get approval of more players), it takes fewer non-veto players to defend themselves; as such, smaller groups are sufficient. Conversely, the largest group (all non-veto players together) is formed when a single vote from a non-veto player is sufficient for veto players to accept a redistribution; in this case, non-veto players can only keep a positive payoff by holding equal amounts.

Now consider the number of groups that (the non-veto part of) the society is divided into. Intuitively, the number of groups corresponds to the maximum possible economic heterogeneity that a society can have in equilibrium. If we interpret the equally endowed non-veto members of the society as economic classes (in the sense that members of the same class have similar possessions, whereas members of different classes have different amounts of wealth, despite having the same political power), then the number of groups would correspond to the largest number of economic classes that the society can contain. With this interpretation, Proposition 4 implies that it is members of the same or richer economic classes that protect a non-veto player from expropriation. Still, there might be some residual indeterminacy about the number of classes: for any parameters it is possible that all non-veto players possess zero and thus belong to the same class; similarly, the characterization in Proposition 3 allows for classes that are larger than others and that span several groups G_j. Thus, societies with few groups are bound to be homogenous (among non-veto players), whereas societies with many veto groups might be heterogenous with respect to wealth.

To better understand the determinants of the number of groups, take n large and v small (so that m is large enough) and start with the smallest possible value of $k = v + 1$. Then all the non-veto players possess the same wealth in any equilibrium. In other words, all players, except perhaps those endowed with veto power, must be equal. If we increase k, then two groups will form, one of which may possess a positive amount, while the rest possesses zero, which is clearly more heterogenous than for $k = v + 1$. If we increase k further beyond $v + (m + 1)/2$, then both groups may possess positive amounts and a third group will form further, and so forth. In other words, as k increases, so does the number of groups, which implies that the society becomes less and less homogenous and can support more and more groups of smaller size. We see that in this model, heterogeneity of the society is directly linked to difficulty of expropriation, measured by the degree of majority needed for expropriation or, equivalently, by the minimal size of a coalition that is able to resist attempts to expropriate. If we interpret the equally endowed groups as economic classes, then we have the following result: the more politically difficult it is to expropriate, the finer is the class division of the society.

COROLLARY 3: *Suppose that* $k = v + 1$; *as before*, $d = n - v$. *In this case*, *an allocation* x *is stable if* $x_i = x_j$ *for all non-veto players* i *and* j, *that is*, *if all non-veto players hold the same amount*. *More generally*, *a single group of non-veto players with a positive amount of wealth may be formed if and only if* $k - v \equiv q \leq (m + 1)/2$. *In this case*, *some* $n - k + 1$ *non-veto players belong to the group and get the same amount*, *and the rest get zero*.

Proposition 5 dealt with comparing stable allocations for different k and v. We now study whether or not an allocation that was stable under some rules k and v remains stable if these rules change. For example, suppose that we make an extra individual a veto player (increase v) or increase the majority rule requirement (increase k). A naive intuition would say that in both these cases, individuals would not be worse off from better property rights protection. As the next proposition shows, in general, the opposite is likely to be true. Let $\mathbf{S}_{k,V}$ denote the set of stable allocations under the supermajority requirement k and the set of veto players V.

PROPOSITION 6: *Suppose that allocation* x *is stable for* k ($k < n$) *and* v ($x \in \mathbf{S}_{k,V}$). *Then the following statements hold*:

(i) *If we increase the number of veto players by granting an individual* $i \notin V$ *veto power so that the new veto set is* $V \cup \{i\}$, *then allocation* $x \in \mathbf{S}_{k,V\cup\{i\}}$ *if and only if* $x_i = 0$.

(ii) *Suppose* $k + 1 < n$ *and all groups* G_j, $j \geq 0$, *had different amounts of wealth under* x: $x_{G_j} \neq x_{G_{j'}}$ *for* $j' \neq j$ (*and* $x|_M \neq 0$). *If we increase the majority requirement from* k *to* $k' = k + 1$, *and* $k' < n$, *then* $x \notin \mathbf{S}_{k+1,V}$.

The first part of this proposition suggests that adding a veto player makes an allocation unstable, and therefore will lead to a redistribution that hurts some individual. There is only one exception to this rule: if the new veto player had nothing to begin with, then the allocation will remain stable. On the other hand, if the new veto player had a positive amount of wealth, then although he will be weakly better off from becoming a veto player, there will be at least one other non-veto player who will be worse off. Indeed, removing a member of one of the groups G_j without changing the required sizes of the groups *must* lead to redistribution. This logic would not apply if $V' = N$, when all players become veto players; however, the proposition is still true in this case because then i would have to be the last non-veto player, and under $k < n$ he would have to get $x_i = 0$ in a stable allocation x. Interestingly, removing a veto player i (making him non-veto) will also make x unstable as long as $x_i > 0$. This is, of course, less surprising, as this individual may be expected to be worse off.

The second part says that if all groups got different allocations (which is the typical case), then an increase in k would decrease the required group sizes, leading to redistribution. When some groups have equal amounts of wealth in a stable allocation, then allocation x may, in principle, remain stable. This is trivially true when all non-veto players get zero ($x_i = 0$ for all $i \notin V$), but, as the following Example 7 demonstrates, this is possible in other cases as well.

EXAMPLE 7: Suppose $n = 7$, $V = \{\#7\}$, $b = 6$, and the supermajority requirement is $k = 5$. Then $x = (1, 1, 1, 1, 1, 1; 0)$ is a stable allocation, because $d = 3$ and the non-veto players form two groups of size three. If we increase k to $k' = 6$, then x remains stable, as then $d' = 2$ and x has three groups of size two.

5. DISCUSSION

In this section, we put two main contributions of our paper—the emergence of a class structure in a multilateral bargaining setting and the non-monotonic effect of the number of veto players and supermajority requirements on the stability of allocations—in the context of the existing literature. Also, we discuss the role of specific technical assumptions.

In Propositions 2 and 3, we established one-to-one correspondence between stable allocations of the non-cooperative bargaining game and a unique von Neumann–Morgenstern stable set, which greatly simplified the analysis. Similar links between cooperative and non-cooperative definitions of stability were observed in earlier works: the theoretical foundations for implementation of the vNM-stable set in non-cooperative games are laid down in Anesi (2006, 2010) and Acemoglu, Egorov, and Sonin (2012), in games of different generality. In contrast to these studies, we allow players to be indifferent among allocations, which required us to define vNM stability with respect to a different dominance relation. The main novel aspect of the current paper is the explicit and intuitive characterization of the stable set (Proposition 3). This characterization allowed us to more thoroughly explore the forces that make a stable allocation stable and to study reactions of these stable allocations to exogenous shocks, thus identifying players who would resist deviations from a stable allocation (Proposition 4).

The tractability of the model, made possible by this explicit characterization, allowed to study comparative statics with respect to the two main parameters: the number of veto players and the supermajority requirement. In static models, more veto players and/or a higher degree of supermajority make any given allocation more likely to be stable, because a larger coalition is required to change it (see, e.g., Tsebelis (2002) in the case of veto players and Chapter 6 in Austen-Smith and Banks (2005) in the case of supermajority requirements). This paper proves that in dynamic models the impact of these parameters on stability of allocations is nonmonotone, and it is the first do so, to the best of our knowledge. We also show, in Proposition 6, that an increase in the number of veto players or the supermajority requirement generically destroys stability of an allocation.

While the idea of nonmonotonicity in a multilateral bargaining setting is intuitive, such results have not been stated formally, most likely due to the difficulty of obtaining a tractable characterization in such models. However, similar effects in the literature on voting on reforms (even in two-period models) have been known. In Barbera and Jackson (2004), if some voting rule is stable, then one that requires a larger degree of supermajority is not necessarily stable, because while more votes are needed to change the rule, many more players might find the new rule suboptimal and be willing to change it. Similarly, Gehlbach and Malesky (2010) show that an additional veto player might allow for a reform that would have been impossible otherwise as some players fear a slippery slope.[14]

The explicit characterization demonstrates that stable allocations are organized as "economic classes," members of which protect each other from expropriation. This is in contrast to the existing literature on bargaining with an endogenous status quo, starting with Kalandrakis (2004), which emphasizes eventual appropriation of the entire surplus by a single player. We would argue that economic classes comprised of similar individuals is a more realistic outcome. The observation that different *ex ante* identical players might be split into groups with similar payoffs has remote antecedents in the legislative bargaining literature. For example, in Baron and Ferejohn (1989), the set of players is ultimately subdivided into three distinct groups, ordered in terms of wealth: the proposer, the winning coalition, and the rest.[15] In Bernheim, Rangel, and Rayo (2006), the last proposer is able to implement his ideal policy, thus again dividing the society into three unequal groups. In these papers, this split into groups resulted from terminal-period effects. Our results demonstrate that economic classes may emerge in a dynamic environment with no terminal period; we also study the effects of the models' primitives on their numbers and their sizes, showing, in particular, that a larger supermajority requirement results in a larger number of smaller classes (Propositions 3 and 5).

Any model of multilateral bargaining makes a number of specific modeling assumptions.[16] Perhaps most consequentially, ours is a model of discrete policy space. Overall, the literature on multilateral bargaining with endogenous status quo is split between papers that assume a continuous (divide-a-dollar) policy space and a discrete (e.g., finite) one. Baron and Ferejohn (1989), Kalandrakis (2004, 2010), Baron and Bowen (2015), Richter (2014), Anesi and Seidmann (2014), and Nunnari (2016), among others, assume that the policy space is continuous, while Anesi (2010), Diermeier and Fong (2011, 2012), and Anesi and Duggan (2017) assume a discrete one, as we do. We view the benefit of our approach mainly in that it considerably simplifies the analysis: in fact, the use of the von Neumann–Morgenstern-stable set in all voting models that we are aware of requires a discrete space. While we are not able to analyze the model with a continuous policy space, it is reassuring that the limit set of our equilibrium allocations when the size of the unit approaches zero has the same class structure as the set of stable sets in Proposition 3, suggesting further robustness of our results.

When indifferences are present because of the nature of the model, most papers, including Kalandrakis (2004), Diermeier and Fong (2011), and Anesi and Duggan (2015), assume that a player supports the new proposal when he is indifferent. In contrast, Baron and Bowen (2015) argue that it is important to assume that players vote against the proposal when they are indifferent. Anesi and Seidmann (2015) assume that players are supportive of the proposal when they are indifferent, depending on the coalition formed on the equilibrium path. (Anesi and Duggan (2015) extend this construction to the spatial setting.) We assume that transitions unlock an arbitrarily small budget that may be

[14]In models with information aggregation in voting (e.g., Feddersen and Pesendorfer (1998)), the supermajority requirement may have nonmonotone effects as it influences pivotal events that players condition upon.

[15]While the identity of the first proposer and thus the realized allocation is random, the expected payoffs are identical in all subgame perfect equilibria, as shown by Eraslan (2002) and, in a more general setting, by Eraslan and McLennan (2013).

[16]There is an important parallel in the coalition formation literature. See, for example, Seidmann and Winter (1998) on the impact of the possibility of renegotiation on the structure of the ultimate coalition, Hyndman and Ray (2007) on equilibria in games with possible binding constraints, or Ray and Vohra (2015) on the farsighted stable set.

used to resolve indifferences. Intuitively, this breaks indifferences in the direction of accepting the proposal, which is consistent with the contract theory literature Bolton and Dewatripont (2004). The fact that the results hold for any size of this additional budget provided that it is small enough points to robustness of our equilibria.

6. CONCLUSION

The modern literature often considers constitutional constraints and other formal institutions as instruments of property rights protection. The relationship between veto power given to different government bodies, supermajority requirements, or additional checks and balances and better protection seems so obvious that there is little left to explain. Allston and Mueller (2008) proclaim, "A set of universally shared beliefs in a system of checks and balances is what separates populist democracies from democracies with respect for the rule of law." Yet, from a political economy perspective, property rights systems should be understood as equilibrium outcomes rather than exogenous fixed constraints. Legislators or, more generally, any political actors, cannot commit to entitlements, prerogatives, and rights. Rather, any allocation must be maintained in equilibrium.

Our results suggest that a dynamic perspective may lead to a more subtle understanding of the effects of veto players and supermajority rules. In a dynamic environment, they lead to emergence of endogenous veto groups of players that sustain a stable allocation in equilibrium. The society has a "class structure": any non-veto player with a positive wealth is part of a group of equally endowed individuals who have incentives to protect each other's interests. The effect of exogenous constraints on endogenous veto groups is complex. One the one hand, endogenous veto groups may protect each other in equilibrium even in the absence of formal veto rights. One the other hand, adding more veto players may lead to more instability and policy change if such additions upset dynamic equilibria where players were mutually protecting each other.

Models of multilateral bargaining with endogenous status quo seem to be a natural and very fruitful approach to study the political economy of property rights protection. Our results point to the importance of looking beyond formally defined property rights and, more generally, beyond formal institutions. Thus, a change in formal institutions might strengthen protection of property rights of designated players, yet have negative consequences for protection of property rights of the others, and, as a result, have a negative overall effect.

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SUPPLEMENT TO "POLITICAL ECONOMY OF REDISTRIBUTION"

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BY DANIEL DIERMEIER, GEORGY EGOROV, AND KONSTANTIN SONIN

A1. PROOFS

WE START WITH A FEW AUXILIARY LEMMAS that help us prove Proposition 1. In what follows, we let ξ_i^t denote transfers less transition costs, if any, obtained by player i in period t.

LEMMA A1: *Any protocol-free MPE σ is acyclic.*

PROOF: Let $\phi = \phi_\sigma$ be the equilibrium transition mapping generated by equilibrium σ. Suppose that there is a cycle starting from x: $\phi(x) \neq x$, but $\phi^l(x) = x$ for some $l > 1$. Without loss of generality, let l be the minimal such value, that is, the length of the cycle. Let us first show that for every $i \in V$, $[\phi^j(x)]_i = x_i$ for all j. Suppose not. Then without loss of generality we may assume to have chosen x such that $x_i \geq [\phi^j(x)]_i$ for all j (so i gets his maximum allocation along the cycle) and, moreover, that $[\phi(x)]_i < x_i$. Then, in the period that started with $x^{t-1} = x$ and where, in equilibrium, transition to $\phi_\sigma(x)$ is made, the continuation utility of player i satisfies (after taking the expectation over possible realizations of the protocols)

$$U_i^t \leq \left[\phi_\sigma(x)\right]_i + \xi + \beta\left(\left[\phi_\sigma^2(x)\right]_i + \xi\right) + \cdots + \beta^{l-1}\left(\left[\phi_\sigma^l(x)\right]_i + \xi\right) + \beta^l U_i^t,$$

where $\xi \in [0, (b+1)\varepsilon]$ is the maximum possible value of ξ_i^t over different periods. We thus have

$$\begin{aligned} U_i^t &\leq \frac{\left[\phi_\sigma(x)\right]_i + \xi + \beta\left(\left[\phi_\sigma^2(x)\right]_i + \xi\right) + \cdots + \beta^{l-1}\left(\left[\phi_\sigma^l(x)\right]_i + \xi\right)}{1-\beta^l} \\ &\leq \frac{(x_i - 1) + \xi + \beta(x_i + \xi) + \cdots + \beta^{l-1}(x_i + \xi)}{1-\beta^l} \\ &= \frac{x_i + \xi}{1-\beta} - \frac{1}{1-\beta^l} < \frac{x_i + \xi}{1-\beta} - 1. \end{aligned}$$

At the same time, if player i always vetoes all proposals in all subsequent periods, his continuation utility would be $\tilde{U}_i^t = \frac{x_i}{1-\beta}$. Since $\frac{\xi}{1-\beta} < \frac{(b+1)\varepsilon}{1-\beta} < 1$, we have $U_i^t < \tilde{U}_i^t$, which implies that player i has a profitable deviation. Hence, it must be that $[\phi_\sigma^j(x)]_i = x_i$ for all $j \geq 1$ and for all $i \in V$.

Since each veto player gets x_i in each period, the equilibrium payoff of each player must equal $U_i^t = \frac{x_i - \delta}{1-\beta}$. However, player i can always guarantee himself $\tilde{U}_i^t = \frac{x_i}{1-\beta}$ by vetoing all proposals. Therefore, he has a profitable deviation, which is impossible in equilibrium. This contradiction completes the proof. *Q.E.D.*

LEMMA A2: *Consider a one-step mapping ϕ, which is independent of protocols, and suppose that the current period is t and the current allocation is $x = x^{t-1}$. Suppose that some*

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player i has $[\phi(y)]_i > [\phi(z)]_i$ *for some* $y, z \in \mathbf{A}$. *Then player i prefers transition to y to transition to z; in other words (expectations are with respect to realization of protocols),*

$$y_i + \mathbb{E}\xi_i^t + \sum_{\tau=1}^{\infty} \beta^\tau \big([\phi(y)]_i + \mathbb{E}\xi_i^{t+\tau} \big) > z_i + \mathbb{E}\tilde{\xi}_i^t + \sum_{\tau=1}^{\infty} \beta^\tau \big([\phi(z)]_i + \mathbb{E}\tilde{\xi}_i^{t+\tau} \big), \tag{A1}$$

where ξ *and* $\tilde{\xi}$ *reflect the transfers on path that follow acceptance of y and z, respectively. Furthermore, the same is true if* $[\phi(y)]_i = [\phi(z)]_i$, *but* $y_i > z_i$.

PROOF: Suppose $[\phi(y)]_i > [\phi(z)]_i$, but the inequality (A1) does not hold. Since $\xi_i^{t+\tau}, \tilde{\xi}_i^{t+\tau} \in [0, (b+1)\varepsilon]$ for any $\tau \geq 0$, this must imply

$$y_i + \sum_{\tau=1}^{\infty} \beta^\tau [\phi(y)]_i \leq z_i + \sum_{\tau=1}^{\infty} \beta^\tau [\phi(z)]_i + \frac{(b+1)\varepsilon}{1-\beta}. \tag{A2}$$

Since $[\phi(y)]_i > [\phi(z)]_i$ implies $[\phi(y)]_i - [\phi(z)]_i \geq 1$, this implies

$$y_i + \frac{\beta}{1-\beta} \leq z_i + \frac{(b+1)\varepsilon}{1-\beta}.$$

Given that $z_i - y_i \leq b$, this implies $\frac{\beta-(b+1)\varepsilon}{1-\beta} \leq b$, which, since we assumed $(b+1)\varepsilon < 1-\beta$, implies $\frac{\beta}{1-\beta} \leq b+1$, which is equivalent to $\beta \leq 1 - \frac{1}{b+2}$, a contradiction. This proves the first part of the lemma.

Now suppose that $[\phi(y)]_i = [\phi(z)]_i$, but $y_i > z_i$. As before, assume not, in which case (A2) would hold. Now, given that $y_i - z_i \geq 1$, (A2) would imply $1 \leq \frac{(b+1)\varepsilon}{1-\beta}$, which contradicts our assumption that $(b+1)\varepsilon < 1-\beta$. This contradiction completes the proof. *Q.E.D.*

LEMMA A3: *Suppose that in protocol-free MPE* σ, $x \in \mathbf{A}$ *is such that* $x \neq \phi_\sigma(x) = \phi_\sigma^2(x)$. *Then* $\phi_\sigma(x) \rhd x$.

PROOF: Denote $y = \phi_\sigma(x)$. Let us first prove that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$. Suppose, to obtain a contradiction, that this is not the case. Take some veto player l and consider protocol π where only player l proposes and does so only once (so $\pi = (l)$). Under this protocol, alternative y must be proposed and subsequently supported at the voting stage by a winning coalition of players. Now consider any agent i such that $y_i < x_i$, which implies $x_i - y_i \geq 1$. If y_i is accepted, agent i gets continuation utility (assuming the current period is t) that satisfies

$$U_i^t \leq y_i + (b+1)\varepsilon + \beta\big(y_i + (b+1)\varepsilon\big) + \cdots = \frac{y_i + (b+1)\varepsilon}{1-\beta}.$$

If, however, y_i is rejected, then the continuation utility satisfies

$$\tilde{U}_i^t \geq x_i + \beta y_i + \beta^2 y_i + \cdots = x_i + \frac{\beta}{1-\beta} y_i.$$

Since $b\varepsilon < 1-\beta$, we have

$$\begin{aligned} U_i^t - \tilde{U}_i^t &\leq \frac{y_i + (b+1)\varepsilon}{1-\beta} - \left(x_i + \frac{\beta}{1-\beta} y_i\right) \\ &= y_i - x_i + \frac{(b+1)\varepsilon}{1-\beta} \leq \frac{(b+1)\varepsilon}{1-\beta} - 1 < 0. \end{aligned}$$

Therefore, such player i prefers the alternative y to fail at the voting stage. This implies that $U_i^t - \tilde{U}_i^t \geq 0$ is possible only if $y_i \geq x_i$, and, by assertion, the set of such players does not form a winning coalition, which means that y cannot be accepted at this voting stage. This contradicts that σ is equilibrium, which proves that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$.

It remains to prove that for some $i \in V$, $y_i > x_i$ and $\|\phi_\sigma(x)\| \leq \|x\|$. Both results immediately follow from the fact that transition to $\phi_\sigma(x)$ is feasible and is not blocked by any veto player because of transition cost. Now, by definition of the binary relation $\rhd$, we have $\phi_\sigma(x) \rhd x$, which completes the proof. *Q.E.D.*

LEMMA A4: *Every protocol-free equilibrium is simple, that is, for every* $x \in \mathbf{A}$, $\phi_\sigma^j(x) = \phi_\sigma(x)$ *for all* $j \geq 1$.

PROOF: Suppose that there is a protocol-free equilibrium σ that is not simple, which means that there is $x \in \mathbf{A}$ such that $\phi_\sigma^2(x) \neq \phi_\sigma(x)$. By Lemma A1, σ is acyclic and, therefore, the path starting from x, $\phi_\sigma(x)$, $\phi_\sigma^2(x), \ldots$, stabilizes after no more than $|\mathbf{A}|$ iterations, and thus its limit $\phi_\sigma^\infty(x) = \phi_\sigma^{|\mathbf{A}|}(x)$ is well defined. Denote the set of all such $x \in \mathbf{A}$ by Y, so

$$Y = \{x \in \mathbf{A} : \phi_\sigma^2(x) \neq \phi_\sigma(x)\} \neq \emptyset.$$

Take allocation $y \in Y$ such that $\phi_\sigma^\infty(y) = \phi_\sigma^2(y)$ (notice that such y exists: indeed, if we take any $x \in Y$ and the minimal number such that $\phi_\sigma^\infty(x) = \phi_\sigma^j(x)$ is $j > 2$, then we can take $y = \phi_\sigma^{j-2}(x)$). Notice that we must have $\sum_{i \in V}[\phi_\sigma^2(y)]_i > \sum_{i \in V}[\phi_\sigma(y)]_i$, for otherwise the transition from $\phi_\sigma(y)$ to $\phi_\sigma^2(y)$ would be blocked by some veto player due to the cost of transition.

Consider veto player l for whom $[\phi_\sigma^2(y)]_l > [\phi_\sigma(y)]_l$. Suppose that in period t where the status quo is y, protocol $\pi = (l)$ is realized. Since σ is protocol-free, this must imply that player l proposes alternative allocation $\phi_\sigma(y)$ and some feasible transfers ξ, and this proposal is subsequently accepted. Now suppose that protocol $\pi' = (l, l)$ is realized and suppose that the game reached the second stage of the protocol. This subgame is isomorphic to one where protocol π has just been realized; consequently, in equilibrium, it must be that $\phi_\sigma(y)$ is proposed, accompanied with transfer ξ, and is accepted.

Let us prove that if in the second stage, the society decides to move to $\phi_\sigma(y)$, then in the first stage player l would be better off proposing $\phi_\sigma^2(y)$ and some feasible vector of transfers $\tilde{\xi}$, which would be accepted. Notice that in the following period, a transition from $\phi_\sigma(y)$ to $\phi_\sigma^2(y)$ would take place, which means that each player would receive a certain expected vector of transfers ξ. On the other hand, if transition to $\phi_\sigma^2(y)$ takes place in the current period, then the next period would have no transition, and in expectation, each veto player would get a transfer $\frac{\varepsilon}{v}$ (since each of them is equally likely to be the last player, who would be able to get the entire budget ε with probability 1). Notice that

$$\|\xi\|+\|\tilde{\xi}\| \le \left(\max\left(\sum_{i\in V}\left[\phi_\sigma(y)\right]_i-\sum_{i\in V}y_i,0\right)+1\right)\varepsilon$$
$$+\left(\max\left(\sum_{i\in V}\left[\phi_\sigma^2(y)\right]_i-\sum_{i\in V}\left[\phi_\sigma(y)\right]_i,0\right)+1\right)\varepsilon$$
$$\le\left(\sum_{i\in V}\left[\phi_\sigma^2(y)\right]_i-\sum_{i\in V}y_i+2\right)\varepsilon.$$

Take some small value $\alpha>0$ and define vector χ by $\chi_i=\xi_i+\tilde{\xi}_i-\frac{\varepsilon}{v}\mathbf{I}\{i\in V\}+\alpha_i$, where $\alpha_i=-\alpha$ for $i\neq l$ and $\alpha_l=(n-1)l$. Then $\|\chi\|\le(\sum_{i\in V}[\phi_\sigma^2(y)]_i-\sum_{i\in V}y_i+1)\varepsilon$ and $\chi_i\ge 0$ for all i (for $i=l$ this is true because $\xi_l>0$), so χ is a feasible vector if a transition to $\phi_\sigma^2(y)$ is proposed. If player l proposes such a transition to $\phi_\sigma^2(y)$ and offers feasible vector χ, then all players $i\in N$ who have $[\phi_\sigma^2(y)]_i\ge[\phi_\sigma(y)]_i$ must prefer such a transition to $\phi_\sigma^2(y)$ to happen rather than not. But since the transition from $\phi_\sigma(y)$ to $\phi_\sigma^2(y)$ would happen in a period starting with $\phi_\sigma(y)$, Lemma A3 implies $\phi_\sigma^2(y)\rhd\phi_\sigma(y)$, but this implies that the set of players who are better off if $\phi_\sigma^2(y)$ is accepted at the first stage is a winning coalition. This means that $\phi_\sigma^2(y)$ would be accepted if proposed, which implies that player l has a profitable deviation. This is a contradiction that completes the proof. *Q.E.D.*

LEMMA A5: *If σ is a simple protocol-free MPE, then for all $x\in\mathbf{A}$ either $\phi_\sigma(x)=x$ or $\phi_\sigma(x)\rhd x$.*

PROOF: By Lemma A1, σ is acyclic, and by Lemma A4, it is simple. Then for any $x\in\mathbf{A}$, we must have $\phi_\sigma^2(x)=\phi_\sigma(x)$. Now if $\phi_\sigma(x)=x$, the result is automatically true, and if $\phi_\sigma(x)\neq x$, then it follows immediately from Lemma A3. *Q.E.D.*

LEMMA A6: *Suppose that protocol-free MPE σ is played, and suppose that in period t, $x^{t-1}=x$. Then if there exists $y\in\mathbf{A}$ such that $\phi_\sigma(y)=y$ and $y\rhd x$, then x cannot be stable: $\phi_\sigma(x)\neq x$.*

PROOF: Suppose, to obtain a contradiction, that $\phi_\sigma(x)=x$. Let l be a veto player such that $y_l>x_l$ (such l exists as $y\rhd x$). Consider protocol $\pi=(l)$ (or any protocol ending with l). If a proposal made in this period is rejected, then each player i gets $\tilde{U}_i^t=\frac{x_i}{1-\beta}+\frac{\beta}{1-\beta}\frac{\varepsilon}{v}\mathbf{I}\{i\in V\}$.

Suppose player l makes proposal (y,ξ), where $\xi_i=\frac{(\|y\|-\|x\|+1)\varepsilon-\delta}{n}$. Since $\|y\|-\|x\|\ge 1$ and $\delta<\varepsilon$, we have $\xi_i\ge 0$ for all $i\in N$, so ξ is a feasible transfer. This means that each player i for which $y_i\ge x_i$ would get $\frac{y_i}{1-\beta}+\xi_i+\frac{\beta}{1-\beta}\frac{\varepsilon}{v}\mathbf{I}\{i\in V\}$ if the proposal is accepted, which exceeds $\tilde{U}_i^t$ that he would get if the proposal is rejected. Since $y\rhd x$, such players form a winning coalition, which implies that the proposal (y,ξ) would be accepted if made. Then player l has a profitable deviation, which is impossible. This contradiction completes the proof. *Q.E.D.*

PROOF OF PROPOSITION 2: *Part 1.* Take any simple protocol-free MPE σ and let $\mathbf{S}_\sigma=\{x\in\mathbf{A}:\phi_\sigma(x)=x\}$. By Lemma A1, it is nonempty. Let us prove that it satisfies internal stability. Suppose that for some $x,y\in\mathbf{S}_\sigma$, we have $y\rhd x$. Then by Lemma A6, $\phi_\sigma(y)=$

y implies $\phi_\sigma(x) \neq x$, which contradicts that $x \in \mathbf{S}_\sigma$. This contradiction proves that $\mathbf{S}_\sigma$ satisfies internal stability.

Let us now show that $\mathbf{S}_\sigma$ satisfies external stability. Take $x \notin \mathbf{S}_\sigma$. Then by Lemma A5, $\phi_\sigma(x) \rhd x$. Since σ is simple, $\phi_\sigma(x) \in \mathbf{S}_\sigma$, which shows that there exists $y \in \mathbf{S}_\sigma$ such that $y \rhd x$. This proves that $\mathbf{S}_\sigma$ satisfies external stability. This proves that $\mathbf{S}_\sigma$ is von Neumann–Morgenstern-stable set. *Q.E.D.*

LEMMA A7: *If σ is a protocol-free MPE, then $\|\phi_\sigma(x)\| = \|x\|$ for all $x \in \mathbf{A}$.*

PROOF: Suppose not. Then there exists $x \in \mathbf{A}$ for which $\|\phi_\sigma(x)\| < \|x\|$. Since σ is simple by Lemma A4, we have $\phi_\sigma(x) \in \mathbf{S}$. Take some veto player l and consider the protocol $\pi = (l)$; at this stage, player l must propose $\phi_\sigma(x)$ and it must be accepted. Notice, however, that player l may propose allocation y that has $y_l = [\phi_\sigma(x)]_l + 1$ and $y_i = [\phi_\sigma(x)]_i$ for all $i \neq l$, and split the extra ε of available transfers equally among players. By Proposition 3, such allocation y is stable as well. Consequently, all players would be strictly better off from proposal y (with the corresponding transfers) than the equilibrium proposal $\phi_\sigma(x)$. Thus, if a winning coalition was weakly better off from supporting $\phi_\sigma(x)$, it is strictly better off supporting y. Thus, player l has a profitable deviation at the proposing stage, which is a contradiction that completes the proof. *Q.E.D.*

PROOF OF PROPOSITION 1: *Part* (i). Consider the unique von Neumann–Morgenstern-stable set for dominance relation $\rhd$, $\mathbf{S}$ (its existence and uniqueness follow from Proposition 3 proven in the main text). Take any mapping ϕ such that $\phi(x) = x$ for any $x \in \mathbf{S}$ and for any $x \notin \mathbf{S}$, $\phi(x) \in \mathbf{S}$ and $\phi(x) \rhd x$ (the existence of such a mapping follows from external stability of mapping $\mathbf{S}$ implying that for any $\mathbf{S}$, we can pick such $\phi(x) \in \mathbf{S}$) and, moreover, $\|\phi(x)\| = \|x\|$ (the existence of such ϕ follows from Proposition 3 as well, as otherwise one can add $\|x\| - \|\phi(x)\|$ units to some veto player and get an allocation in $\mathbf{S}$ with the required property). Let us prove the following (stronger) result: there is a protocol-free MPE σ such that $\phi_\sigma = \phi$ (notice that σ will in this case be simple, because $\phi^2 = \phi$.)

We construct equilibrium σ using the following steps. For each possible status quo $x \in \mathbf{A}$ and each protocol $\pi \in \Pi$, we define transfers that each player is supposed to get in that period. We use allocations and these transfer utilities to define continuation utilities. After that, we use these continuation utilities to define strategies players would use for each $x \in \mathbf{A}$ and each $\pi \in \Pi$. We then check that under these strategies, players indeed get the transfers that we defined, and no player has a one-shot deviation. This would prove that σ is MPE, which would be protocol-free by construction.

If $x \notin \mathbf{S}$, then let $V_x = \{i \in V : [\phi(x)]_i = x_i\}$ and let $v_x = |V_x|$. Furthermore, let $Z = \sum_{i\in V}[\phi(x)]_i - \sum_{i\in V} x_i > 0$. Let $l = \pi_{|\pi|}$ be the last proposer, and define transfers $\xi_i(x, \pi)$ as

$$\xi_i(x,\pi) = \begin{cases} 0 & \text{if } i \notin V_x \cup \{l\}, \\ \beta \dfrac{Z\varepsilon}{(1-\beta)v + \beta v_x} & \text{if } i \in V_x \setminus \{l\}, \\ (Z+1)\varepsilon - \sum_{j\neq l} \xi_i(x,\pi) & \text{if } i = l. \end{cases} \tag{A3}$$

If, however, $x \in \mathbf{S}$, then the transfer is defined as $\xi_l(x, \pi) = \varepsilon$ for $l = \pi_{|\pi|}$ and $\xi_i(x, \pi) = 0$ otherwise. Given these transfers, the continuation utilities (at the beginning of the period, before protocol is realized) are given by

$$V_i(x) = \left[\phi(x)\right]_i + \sum_{\pi \in \Pi} \xi_i(x, \pi) + \frac{\beta}{1-\beta}\left(\left[\phi(x)\right]_i + \frac{\varepsilon}{v}\mathbf{I}\{i \in V\}\right). \tag{A4}$$

Let us now define strategies as follows. Suppose that in period t, the current status quo is $x = x^{t-1}$ and protocol π was realized. To define strategies, consider the game with timing from Section 2, but define payoffs in case transition to some $y \in \mathbf{A}$ and set of transfers ξ is decided upon given by

$$U_i(y, \xi) = y_i + \xi_i + \beta V_i(y)$$

(in other words, consider the game truncated at the end of the period, that is, a finite game, but with payoffs coinciding with continuation payoffs of the original game).

Define strategies by proceeding by backward induction, with a few exceptions. In the last stage, the proposer $\pi_{|\pi|}$ proposes to transfer to $\phi(x)$ (or to stay, if $\phi(x) = x$), and offers transfers $\xi_i(x, \pi)$. We require that all players who are at least indifferent vote for this proposal to pass. If any other proposal is made, as well as in all previous stages, we require that players play any strategies consistent with backward induction, except that we require that players vote *no* when indifferent.

Let us show that the players have no incentive to deviate for any strategy that we defined. The one-shot deviation principle applies, so we need to verify that no player has a profitable deviation at any stage. Now consider the two cases $\phi(x) = x$ and $\phi(x) \neq x$ separately.

First, consider the case $\phi(x) \neq x$. Let us check that at the last stage, it is a best response for any player i with $[\phi(x)]_i \geq x_i$ to accept, which would imply that this proposal is indeed accepted. Indeed, both accepting and rejecting results in getting the same allocation $[\phi(x)]_i$ in two periods; thus, if for some player i, $[\phi(x)]_i > x_i$, then by Lemma A2 he is strictly better off if $\phi(x)$ is accepted. Consider a player i with $[\phi(x)]_i = x_i$. If $i \notin V$, then he gets transfer $\xi_i(x, \pi) = 0$ if $\phi(x)$ is accepted, but he gets the same in the following period if the proposal is rejected, which implies that he is indifferent, so supporting $\phi(x)$ is a best response. If $i \in V_x \setminus \{l\}$, then he gets the transfer $\xi_i(x, \pi)$ if the alternative is accepted, and it makes him exactly indifferent between accepting and rejecting. Finally, if $i \notin V_x$ or $i = l$, the player is strictly willing to accept. Thus, for all veto players, it is a best response to support the alternative. Since $\phi(x) \rhd x$, the set of players with $[\phi(x)]_i \geq x_i$ is a winning coalition. Finally, $\|\phi(x)\| = \|x\|$, so the transition is feasible. Consequently, there are best responses that result in $\phi(x)$ being accepted.

Taking one step back, let us verify that it is a best response for player $l = \pi_{|\pi|}$ to propose $\phi(x)$. First, since he prefers $\phi(x)$ to be accepted rather than rejected, he would only propose an alternative y if this alternative would be accepted at the voting stage. Suppose there is such an alternative; it suffices to prove that proposing it does not make the player l better off. By Lemma A2, if $[\phi(y)]_i < [\phi(x)]_i$ for some player i, then this player would be better off if y is rejected. Consequently, for y to be accepted in equilibrium, it is necessary that $[\phi(y)]_i \geq [\phi(x)]_i$ for a winning coalition of players, in particular, for all veto players $i \in V$.

Let us prove that $[\phi(y)]_i = [\phi(x)]_i$ for all $i \in V$. To do so, suppose it is not the case, meaning that for some $j \in V$, the strict inequality $[\phi(y)]_j > [\phi(x)]_j$ holds. In addition, notice that $\|y\| \leq \|x\|$ since transition to y is feasible, but $\|\phi(y)\| \leq \|y\|$ (because transition to $\phi(y)$ would be feasible) and $\|x\| = \|\phi(x)\|$ (by assumption that transition to $\phi(x)$ does not result in waste). This implies $\|\phi(y)\| \leq \|\phi(x)\|$, which, together with $\{i \in N : [\phi(y)]_i \geq [\phi(x)]_i\} \in \mathcal{W}$ and $[\phi(y)]_j > [\phi(x)]_j$, implies $\phi(y) \rhd \phi(x)$. Since $\phi(x), \phi(y) \in \mathbf{S}$, this contradicts internal stability of $\mathbf{S}$, which proves that $[\phi(y)]_i = [\phi(x)]_i$ for all $i \in V$.

Notice that for the proposer, player $l = \pi_{|\pi|}$, to prefer transition to y to transition to $\phi(x)$, it must be that $y_l = [\phi(y)]_l = [\phi(x)]_l$, for otherwise we would get a contradiction with Lemma A2. Consider two possibilities. If $\phi(y) = y$, then for player l to be better off, he needs to get a larger transfer $\chi_l > \xi_l(x, \pi)$. However, since all other veto players in V_x were indifferent between accepting their transfer $\xi_i(x, \pi)$ and rejecting, they need to get at least this transfer as well; since other players need to get $\chi_i \geq 0$ as well, such deviation cannot be profitable. If, however, $\phi(y) \neq y$, then $\phi(y)$ will be reached in the following period. Notice that for each $i \in V$ it must be that $y_i \geq x_i$, for otherwise this player would block the transition. This means, in particular, that for players in V_x, $x_i = y_i = \phi(x_i) = \phi(y_i)$ holds, and they therefore need discounted transfer $\chi_i^t + \beta \mathbb{E} \chi_i^{t+1} \geq \xi_i(x, \pi) + \beta \frac{\varepsilon}{v}$ so as to be willing to accept. However, since the transfers available over the two periods are capped at $(Z + 1)\varepsilon - \delta$, player l cannot be better off from such deviation. Therefore, proposing $\phi(x)$ at the last stage is a best response.

We now prove that for any proposal z made at the previous stage by player $\pi_{|\pi|-1}$, the set of players who strictly prefer transition to z do not form a winning coalition. Indeed, suppose that it is; then by Lemma A2 it must be that for all $i \in V$, $[\phi(z)]_i = z_i = y_i$, for otherwise we would have $\phi(z) \rhd y$, which would contradict internal stability of $\mathbf{S}$. This implies that $z = \phi(z)$, for otherwise transition from z to $\phi(z)$ would be impossible; furthermore, the set of transfers χ proposed at this stage must coincide with $\xi_i(x, \pi)$. If so, if some player $i \notin V$ strictly prefers transition to z, this implies that $z_i > y_i$ for such a player. However, this would contradict the characterization results from Proposition 3. This shows that it is a best response for at least $n - k + 1$ players to vote against proposal z, which implies that there is an equilibrium in this subgame where it is not accepted. Proceeding by backward induction, we can conclude that there is an equilibrium in this finite game where no proposal is accepted until the last stage, where y is accepted.

Now consider the game with $x \in \mathbf{S}$. We allow any strategies, but require that players vote against the proposal when indifferent. Now, again by backward induction, we can conclude that if a winning coalition strictly prefers to accept some proposal z, then either $\phi(z) \rhd x$, which contradicts internal stability of $\mathbf{S}$, or $[\phi(z)]_i = z_i = x_i$ for all $i \in V$, in which case the veto player $\pi_{|\pi|}$ that is the last to propose is actually worse off because of transition cost. Thus, there is an equilibrium in the finite game where no proposal is accepted, so x remains stable.

Last, it is straightforward to check that if these strategies are played, then in every period, transfers are given by $\xi(x, \pi)$ as defined above, and thus the continuation utilities at the beginning of period with x as the status quo are given by $V(x)$. This means that if these strategies are played in the original game Γ, no player has a one-shot deviation. Since by construction the strategies are Markovian and transitions do not depend on the realization of the protocol, then σ is a protocol-free MPE. Moreover, it is simple and efficient by construction, which completes the proof of existence of such equilibrium.

Part (ii). Follows from Lemma A1.

Part (iii). Follows from Lemma A4.

Part (iv). Follows from Lemma A7. *Q.E.D.*

PROOF OF PROPOSITION 4: *Part* (i). Lemma A5 implies that $\phi(y) \rhd y$; in particular, for each $j \in V$, $[\phi(y)]_j \geq y_j$ and for at least one of them the inequality is strict. Suppose, to obtain a contradiction, that $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| < d - 1$. Then

$|\{j \in M : [\phi(y)]_j < x_j\}| < d$. But we also have that for each $j \in V$, $[\phi(y)]_j \geq x_j$, with at least inequality strict. This means $\phi(y) \rhd x$, which is impossible, given that $x, \phi(y) \in \mathbf{S}$. Now suppose, to obtain a contradiction, that $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| > d-1$. But then for at least d agents $[\phi(y)]_j < y_j$, which contradicts $\phi(y) \rhd y$. This contradiction proves that $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| = d - 1$. It remains to prove that $y_i \leq [\phi(y)]_i < x_i$. Suppose not, that is, either $[\phi(y)]_i < y_i$ or $[\phi(y)]_i \geq x_i$. In the first case, we would have that at least d agents have $[\phi(y)]_j < y_j$, contradicting $\phi(y) \rhd y$. In the second case, $[\phi(y)]_i \geq x_i$, coupled with the already established $|\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\}| = d - 1$, would mean $|\{j \in M : [\phi(y)]_j < x_j\}| = d - 1$, and therefore $\phi(y) \rhd x$. This is impossible, and this contradiction completes the proof.

Part (ii). This proof is similar to the proof of internal stability in the proof of Proposition 3. Denote $\phi(y) = z$. Then $z \rhd y$ and $x, z \in \mathbf{S}$. We know that x and z have the group structure by Proposition 3. Then let the r groups be $G_1, \dots, G_r$ for x and $H_1, \dots, H_r$ for z, respectively. Without loss of generality, we can assume that each set of groups is ordered so that x_{G_j} and z_{H_j} are nonincreasing in j for $1 \leq j \leq r$. Suppose, to obtain a contradiction, that for some agent $i' \in M$ with $x_{i'} \leq y_i < x_i$, $z_{i'} < y_{i'}$. In that case, among the set $\{j \in M : x_j \geq x_i\}$ there are at most $d - 1$ agents with $z_j < y_j$; similarly, among the set $\{j \in M : x_j < x_i\}$ there are at most $d - 1$ agents with $z_j < y_j$.

We can now proceed by induction, similarly to the proof of Proposition 3, and show that $x_{G_j} \leq z_{H_j}$ for all j. Base: suppose not. Then $x_{G_1} > z_{H_1}$, and then $x_{G_1} > z_s$ for all $s \in M$. But this means that for all agents $l \in G_1$, we have $x_l > z_l$; since their total number is d, we get a contradiction. Step: suppose $x_{G_l} \leq z_{H_l}$ for $1 \leq l < j$, and suppose, to obtain a contradiction, that $x_{G_j} > z_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > z_{H_s}$. Consequently, for an agent $i'' \in \bigcup_{l=1}^{j} G_l$ to have $z_{i''} \geq x_{i''}$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)d = d$ agents in $\bigcup_{l=1}^{j} G_l \subset M$, $z_{i''} \geq x_{i''}$ does not hold (denote this set by D). If that is true, it must be that $\bigcup_{l=1}^{j} G_l$ includes all the agents in D, including agents i and i' found earlier, and, in particular, $x_{G_j} \leq y_i < x_i$. But on the other hand, these d agents are not in $\bigcup_{s=1}^{j-1} H_s$. In particular, this implies that for any $i'' \in D$, $z_{i''} < x_{G_j}$, but $x_{i'} \geq x_{G_j}$, which means $z_i < x_{i'}$. But $z_i \geq y_i$ by part (i) of this proposition, so $y_i < x_{i'}$. But this contradicts the way we chose i' to satisfy $x_{i'} \leq y_i < x_i$. This proves that such i' cannot exist, and thus the $d - 1$ agents other than i who are made worse off satisfy $x_j \geq x_i$. *Q.E.D.*

PROOF OF PROPOSITION 5: This result immediately follows from the formulas $m = n - v$, $d = n - k + 1$, and $r = \lfloor m/d \rfloor$, and from Proposition 3. *Q.E.D.*

PROOF OF PROPOSITION 6: *Part* (i). If $k < n$, then $d > 1$. An allocation x is stable only if $|\{j \in M : x_j > 0\}|$ is divisible by d. If x is stable and some agent i with $x_i > 0$ is made a veto agent, then the set $|\{j \in M' : x_j > 0\}| = |\{j \in M : x_j > 0\}| - 1$ and is not divisible by d; thus x becomes unstable. At the same time, if $x_i = 0$, then the group structure for all groups with a positive amount is preserved; thus x remains a stable allocation.

Part (ii). In this case, the size of each group in x is $d > 2$, and every positive amount is possessed by either no players or d non-veto players. If k increases by 1, d decreases by 2. Then allocation x becomes unstable, except for the case $x|_M = 0$. *Q.E.D.*

A2. EXAMPLES

In the examples below, we do not explicitly consider costs of transition and transfers explicitly, as they would complicate the exposition. Unless specified otherwise, each of the examples below may be modified to accommodate such factors.

EXAMPLE A1—If Costs of Transition Are Assumed To Be 0: Suppose $n = 3$, $v = 1$, and $k = 2$, so there are three players, one of them a veto player, and the rule is simple majority rule. Assume for simplicity that there is only 1 unit that initially belongs to a non-veto player (say, player #1), so the initial allocation is $(1, 0; 0)$. Then there would be an equilibrium where the veto player (proposing last) would propose to move the unit from player #1 to player #2 if it belongs to player #1, and then propose to move it the other way around if it belongs to player #2. Such a proposal would then be supported by the veto player and the other player who receives the unit.

To complete the description of strategies, we can also assume that any proposal made at a protocol stage before the last one, except for the proposal to transfer the good to the veto player, would be vetoed by the veto player (he is indifferent anyway). On the other hand, if a proposal to transfer the unit to the veto player is ever made, the two non-veto players vote against this proposal. They both have incentives to do so, because the equilibrium play gives them the unit in possession every other period, which is better than having the unit taken away.

Thus, without transaction costs, it is possible to have cyclic equilibria, which do not seem particularly natural.

EXAMPLE A2—Example Where Non-Veto Player Proposes Last: Suppose $n = 11$, $v = 1$, and $k = 9$, so there are 11 players, one of them a veto player, and the rule requires agreement of 9 players. The size of a minimal blocking coalition is then three. In this case, in any protocol-free MPE (where the last proposal is done by the veto player), allocation $(3, 3, 3, 2, 2, 2, 1, 1, 1, 10; 0)$ is unstable, and, in any equilibrium, it results in a transfer to an allocation where all players except for player #10 (the one possessing 10 units in the beginning of the game) are better off. To simplify the following argument, let us focus on the equilibrium where an immediate transition to $(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$ takes place.

Consider, however, what would happen if a protocol has a non-veto player propose last. Specifically, suppose the protocol has two players: first the veto player (player #11) proposes and then the non-veto player #6 proposes. Consider the last stage and suppose that player #6, instead of proposing to move to $(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$ or to stay in the current allocation $(3, 3, 3, 2, 2, 2, 1, 1, 1, 10; 0)$, proposes to transfer to allocation $(3, 3, 1, 2, 2, 3, 1, 1, 2, 4; 6)$; in other words, in addition to moving some units to the veto player, he also proposes to take 2 units from player #3, and takes 1 unit himself and gives the other one to player #9 in order to "buy" his vote. This proposal leads to a stable allocation, and it makes only two players (player #3 and player #10) worse off. It therefore would be accepted; the veto player would agree, because it gives him 6 of the units right away, and he would be able to get the other 4 the following period. (Notice that player #4 might prefer not to get more units for himself in the short run, out of fear that having 4 or more units in the next period would make him a candidate for complete expropriation.)

Taking one step back and consider the stage where the veto player makes the proposal. He would use the opportunity to get the 10 units belonging to him immediately (which hurts player #10). However, he would not be able to make the society move to

$(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$, which they are supposed to do in equilibrium, because doing so would make players #6 and #9, in addition to #10, worse off, and thus such a proposal would not gather the 9 votes needed to pass. This means that by allowing non-veto players to propose, in some examples we would lose the existence of protocol-free MPE.

This example relies on the fact that non-veto players are not indifferent between different stable allocations, and would want to make the society reallocate the units in their favor. As the results in this paper show, these moves cannot happen in protocol-free equilibria studied in the paper. Consequently, we do not view such a possibility to be natural or robust, and we impose the assumption that non-veto players cannot be the last ones in a protocol to avoid such issues and obtain the existence of protocol-free equilibria.

EXAMPLE A3—Example With Fixed Protocol: Suppose $n = 3$, $v = 1$, and $k = 2$, so there are three players, one of them a veto player, and the rule is simple majority. Consider the allocation $(1, 1; 0)$, where the veto player possesses nothing initially. In a protocol-free equilibrium, this allocation would be stable.

Consider a game where the protocol is fixed at $\pi = (1, 3)$ in each period (we can allow the second player to propose in between the other two and get the same result). We claim that the following transitions are possible in an equilibrium. Player #1 is recognized first, and he proposes to move to $(1, 0; 1)$, which is supported by him and the veto player, and in the following period the veto player gets all the surplus, as usual. If the proposal by player #1 is rejected, however, then player #3 is recognized and proposes to move to $(0, 1; 1)$, and this proposal is supported by himself and player #2. Thus, in equilibrium, the society moves from $(1, 1; 0)$ to $(1, 0; 1)$, and then to $(0, 0; 2)$.

The reason why this example works is the following. Player #1 knows that if he does not promise the veto player a transfer of 1 unit, then he would lose his possession immediately (later the same period), whereas delivering the unit to the veto player allows him to postpone for another period. The veto player knows that he cannot take both units at once (as players #1 and #2 would like to stick to them for another period); however, if he allows player #2 to keep his unit, the latter would not mind participating in expropriation of player #1, because in either case he keeps his unit for the current period and loses it in the following one, along the equilibrium path. Furthermore, if these strategies are played, preserving the status quo $(1, 1; 0)$ is not an option. Thus, there is an equilibrium where non-veto players participate in expropriation of each other.

Notice that this transition (from $(1, 1; 0)$ to $(1, 0; 1)$) cannot arise in a protocol-free equilibrium for the following reason. Suppose the protocol only involves the veto player. In such an equilibrium, he needs to propose to transit to $(1, 0; 1)$. But player #2 will oppose it for obvious reasons, and player #1 would know that if he agrees, then he keeps his unit for one extra period (the current one), but if he rejects, then in protocol-free MPE he faces the same transition to $(1, 0; 1)$ the following period, and thus he would be able to keep the unit for two extra periods, which he obviously prefers. Consequently, such transition would be impossible in this protocol, which contributes to the idea that such transitions are not particularly robust.

EXAMPLE A4—Example of Equilibrium That Is Not Markov Perfect: Suppose $n = 3$, $v = 1$, and $k = 2$, so there are three players, one of them a veto player, and the rule is simple majority. Consider the allocation $(1, 1; 0)$, where the veto player possesses nothing initially.

Suppose that the veto player is always the proposer, so the protocol is $\pi = (3)$. Then the following transitions may be supported in equilibrium. As long as the allocation is

$(1,1;0)$, the veto player proposes to move to $(1,0;1)$ if the period is odd and to move to $(0,1;1)$ if the period is even, and the proposal is supported by him and by the non-veto player who keeps the unit (player #1 in odd periods and player #2 in even periods). Once this transition has taken place, in the following period, the veto player gets everything, thus moving to $(0,0;2)$.

The rationale for non-veto players to support such proposals is that they get to keep their unit for exactly one extra period, regardless of the outcome of the voting. Thus, they are indifferent in such situations, in which case the veto player is able to allocate a small transfer to break this indifference. As a result, there is a SPE where the society moves to $(1,0;1)$ and then to $(0,0;2)$; it is supported by the threat of a move to $(0,1;1)$ (and then again to $(0,0;2)$) if this proposal is rejected.

Two comments are warranted. First, this SPE does not require knowledge of all history, in particular, players' proposals and votes. It only requires that the veto player acts differently in odd and even periods. In particular, this is a dynamic equilibrium (DE) in the sense of Anesi and Seidmann (2015), as if the players are allowed to condition their moves on the past history of alternatives, they of course can make use of the length of this history. Second, such transitions are impossible in a protocol-free equilibrium. Indeed, the proposal to move to $(1,0;1)$ made by the veto player would not be accepted if player 1 knew that the veto player would make this very proposal again in the following period, rather than proposing $(0,1;1)$.

EXAMPLE A5—Example With Random Recognition of Players but Without Protocol-Free Requirement: Suppose $n=5$, $v=2$, and $k=3$, so there are five players, two of them veto players, and the rule is simple majority. Consider the allocation $(1,1,1;0,0)$, where the veto players possess nothing initially. In a protocol-free equilibrium, this allocation would be stable.

Consider a game, where in each period, one player is recognized as the proposer. Furthermore, assume for simplicity that only veto players may be recognized, and each of them is recognized with probability 0.5. Then the following strategies would form a MPE. Suppose that player #4, if he is the agenda-setter, proposes to move to $(2,0,0;1,0)$, and this proposal is supported by the two veto players and player #1. Similarly, if player #5 gets a chance to propose, he proposes to move to $(0,2,0;0,1)$, which is supported by the two veto players and player #2. If either of the proposals is accepted, then in the following period the society moves to $(0,0,0;2,1)$, where the veto players possess everything.

To understand why player #1 supports the transition to $(2,0,0;1,0)$, notice that in this case, he gets payoff 2 in the current period and 0 thereafter. If he rejects, then he keeps 1 in the current period, but in the next period he faces a lottery between 2 and 0, and gets 0 thereafter. His expected continuation payoff is therefore $1+\beta\frac{2+0}{2}=1+\beta<2$. Consequently, he prefers to agree on the transition to $(2,0,0;1,0)$. For the same reason, player #2 would support the transition to $(0,2,0;0,1)$. Notice that neither of the veto players can do better by choosing some other proposal, and therefore these transitions are possible in equilibrium.

Notice that if we impose the requirement that equilibria be protocol-free, which in this case would mean that the transition is the same regardless of the player who gets to make the proposal, such an equilibrium will be ruled out. Thus, the requirement that equilibria do not depend on the protocol is important for our results, but also these equilibria may be considered more robust than the one in this example.

A3. RELATION TO LARGEST CONSISTENT SET

We have proven (Proposition 2) that the set of stable allocations coincides with the vNM-stable set, which is in our case unique. However, as emphasized, for example, in Ray and Vohra (2015), the vNM has the drawback in that it focuses on "static" deviations, that is, those in which a deviating coalition does not foresee the future path of the game. On theoretical grounds, this is a serious objection. One notion to deal with this problem was the largest consistent set, as defined in Chwe (1994). In what follows, we prove that in our setting, the largest consistent set would coincide with a vNM-stable set and thus with the set of stable allocations, that is, the objection concerning farsighted deviation does not apply to our game. In our view, the intuitive reason for this is that in our game, *any* coalition that can make *some* deviation (i.e., a winning coalition) can also make *any* deviation. Coupled with farsightedness (discount factor being high enough), this means that a coalition that would be better off initiating a long path of changing allocations would also be better off transiting immediately to the final allocation in this sequence, and it is also capable of doing so. Thus, allowing for farsighted deviations does not add profitable deviations at states that did not have such deviations. Below we state this result formally.

For any coalition $X \in 2^N \backslash \{\emptyset\}$, define binary relation $\to_X$ on $\mathbf{A}$: for all $x, y \in \mathbf{A}$, $x \to_X y$ if and only if $\|y\| \le \|x\|$ and either $x = y$ or $X \in \mathcal{W}$. In other words, a winning coalition can enforce transition from any x to any y, as long as y contains fewer units, whereas a nonwinning coalition can only preserve the same allocation x. Also, for any coalition $X \in 2^N \backslash \{\emptyset\}$, define binary relation $\prec_X$ on $\mathbf{A}$: for all $x, y \in \mathbf{A}$, $x \prec_X y$ if and only if $X \subset \{i \in N : y_i \ge x_i\}$ and there is $j \in X \cap V$ such that $y_j > x_j$.

We say that x is directly dominated by y, and write $x < y$ if there is coalition X such that $x \to_X y$ and $x \prec_X y$. We say that state x is indirectly dominated by y, and write $x \ll y$ if there exist $x_0, x_1, \dots, x_m \in \mathbf{A}$ such that $x_0 = x$ and $x_m = y$, and $X_0, X_1, \dots, X_{m-1} \in 2^N \backslash \{\emptyset\}$ such that $x_j \to_{S_j} x_{j+1}$ and $x_j \prec_{S_j} y$ for $j = 0, 1, \dots, m-1$. We call a set $Q \subset \mathbf{A}$ consistent if $x \in Q$ if and only if for any $y \in \mathbf{A}$ and any coalition $X \in 2^N \backslash \{\emptyset\}$ such that $x \to_X y$ there exists $z \in Q$ such that $x \nprec_X z$ and either $y = z$ or $y \ll z$. From Chwe (1994), it follows that there is a single largest consistent set, that is, a consistent set P such that for any consistent set Q, $Q \subset P$. We now prove that $P = \mathbf{S}$, that is, the set of stable allocations is the largest consistent set.

PROPOSITION A1: *The set of stable allocations described in Proposition* 3 *is a unique largest consistent set*.

PROOF: First, we need two preliminary observations. First, it is obvious that for any $x, y \in \mathbf{A}$, $x < y$ implies $x \ll y$. In our setup, however, the opposite is also true, so $x < y$ if and only if $x \ll y$. To see this, suppose that $x \ll y$. Take a sequence of states and a sequence of coalitions that establish indirect dominance $x \ll y$. We first notice that $\|x\| = \|x_0\| \ge \|x_1\| \ge \dots \ge \|x_m\| = \|y\|$, so $\|x\| \ge \|y\|$. Furthermore, $x_0 \prec_{X_0} y$ implies $x \ne y$, for otherwise $j \in X_0 \cap V$ such that $y_j > x_j$ would be impossible. Let $l \ge 0$ be the lowest number such that $x_{l+1} \ne x$; it is well defined and satisfies $l < m$. This means that $x \to_{X_l} x_{l+1}$, and since $x_{l+1} \ne x$, it must be that $X_l \in \mathcal{W}$. This also means $x_l \prec_{X_l} y$, and thus $x \prec_{X_l} y$; however, since $X_l \in \mathcal{W}$ and $\|x\| \ge \|y\|$, we have $x \to_{X_l} y$. Now $x \to_{X_l} y$ and $x \prec_{X_l} y$ by definition imply $x < y$, which proves the equivalence.

Second, we prove that $x < y$ if and only if $y \rhd x$. Indeed, suppose $x < y$. Then for some coalition X, $x \rightarrow_X y$ and $x \prec_X y$; the latter implies $x \neq y$, in which case the former implies $X \in \mathcal{W}$. Furthermore, $x \rightarrow_X y$ implies $\|y\| \leq \|x\|$. We also have $X \subset \{i \in N : y_i \geq x_i\}$, and since $X \in \mathcal{W}$, $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$ as well, in which case $V \subset X$, and then $X \cap V$ is nonempty, so there is $j \in V$ such that $y_j > x_j$. This all implies that $y \rhd x$. Conversely, suppose $y \rhd x$. Let $X = \{i \in N : y_i \geq x_i\}$. Then $X \in \mathcal{W}$, and since $y_j > x_j$ for some $j \in V$ and $V \subset X$, then this is true for some $j \in X \cap V$. This implies that $x \prec_X y$. Now $\|y\| \leq \|X\|$ and $X \in \mathcal{W}$ implies $x \rightarrow_X y$; this means that $x < y$.

Let us now show that **S** is consistent.

To show that set **S** is consistent, take any $x \in \mathbf{S}$, and then take any $y \in \mathbf{A}$ and any $X \in 2^N \setminus \{\emptyset\}$ such that $x \rightarrow_X y$. We need to prove that there exists $z \in \mathbf{S}$ such that $x \not\prec_X z$ and either $y = z$ or $y \ll z$. If $y \in \mathbf{S}$, then we can take $z = y$ to satisfy this property, because $x \not\prec_X z$. Indeed, this holds trivially if $x = z$, and otherwise follows by contradiction: if $x \rightarrow_X y$ and $x \prec_X y$, then $x < y$, which implies $y \rhd x$, but for two allocations $x, y \in \mathbf{S}$ this would contradict internal stability by Proposition 3. Thus, consider the case $y \notin \mathbf{S}$. Since $x \in \mathbf{S}$, we have $x \neq y$ and thus $X \in \mathcal{W}$. Take any equilibrium σ and any transition mapping $\phi = \phi_\sigma$, and let $z = \phi(y) \in \mathbf{S}$. Notice that it is impossible that this z satisfies $z = y$, since $y \notin \mathbf{S}$. Furthermore, we must have $x \not\prec_X z$, for otherwise we would again contradict Lemma A6 (because $\|z\| \leq \|y\| \leq \|x\|$ and then $x \prec_X z$ coupled with $X \in \mathcal{W}$ would imply $z \rhd x$). It remains to prove that $y \ll z$, for which it suffices to prove that $z \rhd y$, but this follows immediately from Lemma A5. Thus, $z \in \mathbf{S}$ with the required properties exists.

Now take some $x \notin \mathbf{S}$. Let $y = \phi(x)$(again, $\phi = \phi_\sigma$ for some equilibrium σ) and let $X = \{i \in N : y_i \geq x_i\}$. Then $y \in \mathbf{S} \subset \mathbf{A}$, which implies $y \neq x$; furthermore, $y \rhd x$ by Lemma A5 and thus $X \in \mathcal{W}$. We need to prove that there does not exist $z \in \mathbf{S}$ such that $x \not\prec_X z$ and either $y = z$ or $y \ll z$. Suppose, to obtain a contradiction, that such z exists. Then $z \neq y$, because $x \prec_X y$, which is true since $y \rhd x$. Then we must have $x \not\prec_X z$ and $y \ll z$, and the latter is equivalent to $z \rhd y$. However, this violates internal stability of set **S**, which holds by Proposition 3. This proves that set **S** is consistent.

Let us now show that **S** is the largest consistent set. Suppose, to obtain a contradiction, that set $P \neq \mathbf{S}$ is the largest consistent set; since **S** is consistent, we have $\mathbf{S} \subset P$. As before, take some equilibrium transition mapping ϕ. Take $x \in P \setminus \mathbf{S}$ for which $\|x\|_V = \sum_{j \in V} x_j$ is maximal. Let $y = \phi(x)$ and $X = \{i \in N : y_i \geq x_i\}$. Then $y \in \mathbf{S}$ and $x \prec_X y$. Since P is consistent, there exists $z \in P$ such that $x \not\prec_X z$ and either $y = z$ or $y \ll z$. Notice that $x \not\prec_X z$ implies that $y \neq z$, because $x \prec_X y$. Then $y \ll z$, which is equivalent to $z \rhd y$, but given that $y \in \mathbf{S}$, we must have $z \notin \mathbf{S}$, for otherwise we would get a contradiction to Proposition 3. Thus, $z \in P \setminus \mathbf{S}$, which implies, by the choice of x, that $\|z\|_V \leq \|x\|$. However, we have $y \rhd x$ and $z \rhd y$; thus each $j \in V$ has $z_j \geq y_j \geq x_j$, and for at least one j, one of the inequalities is strict. This implies that $\|z\|_V > \|x\|_V$, a contradiction. This completes the proof that **S** is the largest consistent set. *Q.E.D.*

A4. CHARACTERIZATION FOR $n = 3, 4, 5$

The following tables contain a summary of stable allocations if the number of players is small ($n = 3, 4, 5$). The nontrivial cases, where non-veto players form groups and protect each other, are highlighted.

Number of veto players (v)

Voting rule (k)

n = 3	$v=1$	$v=2$	$v=3$
$k=1$	Non-veto players possess nothing		
$k=2$	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k=3$	All allocations are stable	All allocations are stable	All allocations are stable

Number of veto players (v)

Voting rule (k)

n = 4	$v=1$	$v=2$	$v=3$	$v=4$
$k=1$	Non-veto players possess nothing			
$k=2$	**Non-veto players have equal amounts**	Non-veto players possess nothing		
$k=3$	**Two non-veto players have same amounts, third has nothing**	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k=4$	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable

Number of veto players (v)

Voting rule (k)

n = 5	$v=1$	$v=2$	$v=3$	$v=4$	$v=5$
$k=1$	Non-veto players possess nothing				
$k=2$	**Non-veto players have equal amounts**	Non-veto players possess nothing			
$k=3$	**Three non-veto players have same amounts, fourth has nothing**	**Non-veto players have equal amounts**	Non-veto players possess nothing		
$k=4$	**Non-veto players form either two classes of two or one class of four**	**Two non-veto players have same amounts, third has nothing**	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k=5$	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable

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