Algebra Exam

Solutions and Grading Guide

You should use this grading guide to carefully grade your own exam, trying to be as objective as possible about what score the TAs would give your responses. Full credit is given for correct answers without work; pay attention to the guidance about partial credit for incorrect answers. If you ran out of time, be sure to grade only work you had completed by the time limit.

Don't get caught up in the details – the goal is to help you identify your weak spots and understand how you might perform on the real thing. Grading involves nuance and we can't fully describe every scenario in the grading guidance. There is <u>no credit</u> for work that doesn't show evidence of understanding the problem – you can write a lot and still get no points. The bar for getting 1 point is low – demonstrate you know something about how to solve the problem. The bar for getting most of the points is high – nearly correct, right general approach, with a small error. You should never get full points if you do not have a correct answer. When in doubt, be conservative about how many points you award yourself.

We hope all students will take advantage of this example exam as soon as possible – give yourself time to study if you need to! With that in mind, each section of this document includes some additional guidance about how to think about your performance on the section and where to focus your additional practice.

I. Linear Equations

Of all the sections of the exam, this is probably (1) the most important and foundational and (2) the one that the majority of students know well already. If you did poorly on this section, definitely devote some study time to it, regardless of whether you expect to take the algebra portion of math camp – (1) you'll definitely need it even if you can pass on the strength of the other sections, and (2) since the majority of students come in knowing this material well, we will likely spend less time on it during the math camp algebra sections.

Work	Accepted answers	Grading guidance (if your answer was wrong)
a) $slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{1-2}{8-(-4)} = -\frac{1}{12}$ It's fine if you put the points in the opposite order – you get the same solution: $\frac{2-1}{-4-8} = -\frac{1}{12}$	$-\frac{1}{12}$ Worth 3 pts	1 pt for the correct equation but no evidence of plugging in correctly 1 pt if you did run/rise (-12) or switched the order of the points $\left(\frac{1}{12}\right)$ without providing the equation 2 pts for correct equation and either of the above mistakes
b) Using the slope we just calculated and the equation for point-slope form: $y - y_1 = m(x - x_1)$ $y - 2 = -\frac{1}{12}\big(x - (-4)\big)$ $y - 2 = -\frac{1}{12}(x + 4)$ It's ok to stop here (since the question didn't ask for a specific form). It's also fine if you simplified more: $y - 2 = -\frac{1}{12}x - \frac{1}{3}$ $y = -\frac{1}{12}x + \frac{5}{3}$ You could have also used the other point: $y - y_1 = m(x - x_1)$ $y - 1 = -\frac{1}{12}(x - 8)$ $y - 1 = -\frac{1}{12}x + \frac{2}{3}$ $y = -\frac{1}{12}x + \frac{5}{3}$	Any of these: • $y - 2 = -\frac{1}{12}(x + 4)$ • $y - 2 = -\frac{1}{12}x - \frac{1}{3}$ • $y = -\frac{1}{12}x + \frac{5}{3}$ • $y - 1 = -\frac{1}{12}(x - 8)$ • $y - 1 = -\frac{1}{12}x + \frac{2}{3}$ Notice that the order of terms on each side of the equation is unimportant – for example the first equation could be written:	Add these: 1 pt for the point-slope equation ($y = mx + b$) 1 pt for using the slope in the equation 1 pt for using either of the points in the equation 3 pts if you have a correct equation written in your work but then made a mistake trying to simplify or transcribing it into the solutions box

Work	Accepted answers	Grading guidance (if your answer was wrong)
Notice the final equation is the same either way – proving you have the same equation. It's in the standard slope-intercept form ($y = mx + b$).	$2 - y = -\frac{1}{12}(4 + x)$ Worth 4 pts	
c) You can either remember the distance formula or think of the right triangle with those two points on either end of the hypotenuse and apply the Pythagorean theorem (which is where the distance formula comes from). $distance = \sqrt{\Delta x^2 + \Delta y^2}$ $= \sqrt{(-4-8)^2 + (2-1)^2}$ $= \sqrt{(-12)^2 + 1^2}$ $= \sqrt{145}$	$\sqrt{145}$ Worth 3 pts	2 pts for the correct distance formula or correct numerical setup (if you have down any of the numerical lines shown in the solution) (i.e., you made a simple math error) 1 pt for any evidence of thinking about the Pythagorean theorem or taking the square root of some squared values that were not the correct ones
d) No work required – memorize that it's the negative inverse of the slope.	12 Worth 3 pts	1 pt for either $\frac{1}{12}$ or -12 without work 2 pts if you wrote that it was the negative inverse but didn't calculate correctly (presumably getting either of the above answers) There is a cumbersome way to correctly calculate this manually without having it memorized – combine the Pythagorean theorem with some geometry of right congruent triangles and the slope equation. If you've convinced yourself that you were on the road to doing that, it's worth 1 pt if you got started and 2 pts if you got most of the way there. Not a great strategy for an exam with a time limit ☺ - but certainly worth credit.
e) You should have memorized that parallel lines have identical slopes, so the slope of the line will be $-\frac{1}{12}$. Plug in to the point-slope form. All the notes from	Either $y = \frac{5}{12} - \frac{x}{12}$ Or	1 pt for using the correct slope 1 pt if you wrote the point-slope equation correctly 2 pts for both of the above or if you have the correct

Work	Accepted answers	Grading guidance (if your answer was wrong)
(b) apply in terms of multiple correct answers, but the zero makes some of them redundant. $y - 0 = -\frac{1}{12}(x-5)$ $y = \frac{5}{12} - \frac{x}{12}$	$y = -\frac{1}{12}(x-5)$ Worth 3 pts	equation somewhere but made a math or transcription error getting it to the solution box

II. Systems of Equations & Graphing

Most of the points for this section are in solving systems of equations. This section had some graphing content (which could just as easily have been in the linear equations section) and some geometry. Both are really standalone content areas that just happened to fit into this question on this particular exam. There are just a few techniques you can combine to solve 2 and 3 equation systems. Once you've mastered them, if you're still losing a lot of points it's typically because of straightforward algebra errors. At that point, try slowing down when you're solving these problems, making sure that you double-check all your work, and then practice, practice, practice. If you're still making mistakes, try writing out algebra steps you normally try to do in your head – it's an easy way to catch simple mistakes.

Work	Accepted answers	Grading guidance
a) There are many acceptable ways to solve this. Here is one: Solve the second equation for y: $y = x - 6$ Substitute into the first equation: $8x + 2(x-6) = 8$ Solve: $8x + 2x - 12 = 8$ $10x = 20$ $x = 2$ Plug in to any equation: $y = 2 - 6 = -4$ Here's another: Multiply the second equation by 8 and add: $\begin{array}{r} 8x + 2y = 8 \\ -8x + 8y = -48 \\ \hline 10y = -40 \end{array}$ Solve: $y = -4$	You could write either $x = 2, y = -4$ or $(2, -4)$ Worth 4 pts	1 pt if you solved one equation for a variable and substituted into the other OR if you multiplied one variable by a constant and then added the two equations together such that one variable cancelled out but didn't get to any correct answers 2 pts for getting one correct and the other wrong 3 pts if you used the correct approach throughout but made one small math error

<table>
<tr><td>Substitute into either equation: $-x - 4 = -6$

Solve: $x = 2$</td><td></td><td></td></tr>
<tr><td>b) There are many acceptable ways to solve this. Here is one:

Add the first and third equation: $\begin{array}{r} x + y + z = 6 \\ x - y - z = 0 \\ \hline 2x = 6 \end{array}$

Solve: $x = 3$

Plug in, noticing that the first and third are now redundant:
$\begin{cases} y + z = 3 \\ -2y + 4z = 0 \end{cases}$

Multiply the top by 2 and add: $\begin{array}{r} 2y + 2z = 6 \\ -2y + 4z = 0 \\ \hline 6z = 6 \end{array}$

Solve: $z = 1$

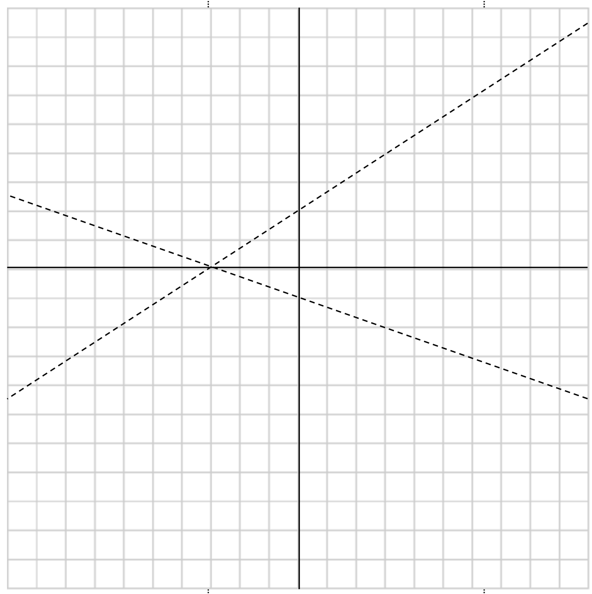
Plug into any of the equations: $3 + y + 1 = 6$

Solve: $y + 4 = 6 \rightarrow \; y = 2$</td><td>You could write either

$x = 3, y = 2, z = 1$

or
$(3,2,1)$

Worth 6 pts</td><td>1 pt if you correctly solved and substituted or added equations such that a variable cancelled once without a correct answer and 3 pts if you did so repeatedly with no correct answers
2 pts for 1 right answer
4 pts for 2 right answers
5 pts if basically correct with one small math error</td></tr>
</table>

c) The equations are both in slope-intercept form, so you should simply read the slope and y-intercept for each line directly from the equations: • You should have one line with a slope of $\frac{2}{3}$ that crosses the y-axis at 2 and the x-axis at -3 • Your other line should have a slope of $-\frac{1}{3}$, cross the y-axis at -1 and the x-axis at -3 You could also have plotted the y-intercept for each line and then calculated the x-intercept by plugging in $y = 0$ to each equation: $0 = \frac{2}{3}x + 2 \;\rightarrow\; x = -2\left(\frac{3}{2}\right) = -3$ $0 = -\frac{1}{3}x - 1 \;\rightarrow\; x = 1\left(-\frac{3}{1}\right) = -3$	 Worth 4 pts	2 pts per line For incorrect lines, you get 1 pt for having either the slope or y-intercept correct
d) Solve the system: $\frac{2}{3}x + 2 = -\frac{1}{3}x - 1 \;\rightarrow\; x = -3$ $y = \frac{2}{3}(-3) + 2 = 0$	We would accept: -3 or $(-3,0)$ or $x = -3$ Worth 3 pts	1 pt IF it's clear your answer came from reading an incorrect graph or from reading a correct graph incorrectly (0 points if that's what happened but there's no evidence on the paper of it) 2 pts if you went through correct steps to solve the equation but made a minor math error
e) $area = \frac{1}{2}(base)(height) = \frac{1}{2}(3)(3) = \frac{9}{2}$	$\frac{9}{2}$ or $4\frac{1}{2}$ or 4.5 Worth 4 pts	1 pt if you tried to add up the boxes on the graph but did it incorrectly (this is a bad approach if the question asks for the precise value) 2 pts for the correct formula with any incorrect

		values or if you tried to calculate and then add together the areas of the two constituent right triangles 2 pts for calculations with a minor error and no equation 3 pts for the correct equation and start to calculations with a minor error

III. Exponents & Logarithms

These questions typically require two things: (1) knowing the appropriate rules of algebra with exponents and logarithms, and (2) being able to break a complicated expression down into several components. If breaking problems like this into parts is challenging for you, start by practicing with much simpler expressions and work up to the types of things you saw on this exam. Spend some time looking at the solutions to see how they're broken down into steps. One trick (that doesn't get written up in most formal explanations because it's based on basic rules of multiplication and division): If you have a complicated fraction, try re-writing it as a series of simple fractions multiplied together. For example:

> $\frac{8w^2x^2y^3}{2x^3yz}$ becomes $\left(\frac{8}{2}\right)\left(\frac{w^2}{1}\right)\left(\frac{x^2}{x^3}\right)\left(\frac{y^3}{y}\right)\left(\frac{1}{z}\right)$, each of which are easy to simplify.
>
> Notice the implied 1 that becomes explicit when we write the w and z terms (in the denominator and numerator, respectively). I've written the 1 in the denominator of the w term to make the point – you would usually leave it out and just write w^2.

For exponents problems, it's often best to start by getting rid of negative exponents (writing the inverse), getting rid of any root symbols (writing $a^{1/2}$ instead of $\sqrt{a}$), and checking if any integers can be re-written as smaller numbers raised to a power (e.g. write 3^3 rather than 27). For logarithm problems, check the base of the logarithm and see whether anything in the problem can be re-written as an exponent with that base.

Work	Accepted answers	Grading guidance
a) $\left(\frac{36}{81}\right)^{-1/2}$	$\frac{3}{2}$	Add these up: 1 pt for handling the negative exponent correctly 1 pt for interpreting the ½ as a square root

Get rid of the negative exponent: $\left(\frac{81}{36}\right)^{1/2}$ Bring in the fractional exponent: $\frac{81^{1/2}}{36^{1/2}}$ It may help to write it as a root: $\frac{\sqrt{81}}{\sqrt{36}}$ And take the roots: $\frac{9}{6}$ Simplify: $\frac{3}{2}$	Worth 4 pts	1 pt for bringing it in correctly and taking the relevant roots 1 pt for fully simplifying the result
b) $\frac{(2x^{-2}y^{3})}{(8x^{5}y^{-8})}$ Break the expression up: $\left(\frac{2}{8}\right)\left(\frac{x^{-2}}{x^{5}}\right)\left(\frac{y^{3}}{y^{-8}}\right)$ Simplify each: $\left(\frac{1}{4}\right)\left(\frac{1}{x^{7}}\right)(y^{11})$ (notice that the negative exponent in the denominator brings the y^{8} into the numerator) You can recombine, but it's not required: $\frac{y^{11}}{4x^{7}}$	Either $$\left(\frac{1}{4}\right)\left(\frac{1}{x^{7}}\right)(y^{11})$$ or $$\frac{y^{11}}{4x^{7}}$$ (the expressions are equivalent) Worth 4 pts	Add these up: 1 pt for simplifying the integers to $\frac{1}{4}$ 1 pt for evidence you handled both negative exponents correctly – either in the form of a correct final expression for the variable or by showing an intermediate step where it is correct (if you made a later mistake combining terms) 1 pt each for correctly adding the relevant exponents for the x and y terms
c) $\log_5(0.2)$ Notice the base of the logarithm is 5. Think about ways to re-write 0.2 so that it includes a 5. $$\log_5\left(\frac{1}{5}\right)$$ Express using an exponent: $\log_5 5^{-1}$ Bring the exponent outside: $-1\log_5 5$ Remember that $\log_a a = 1$, so: -1	$$-1$$ Worth 4 pts	Add these up: 1 pt for re-writing as a fraction involving 5 1 pt for the negative exponent 1 pt for bringing out the exponent 1 pt for the final cancellation
d) $\frac{8^{-1}}{8^{-6}}$	Either $$8^{5}$$ Or 32,768	Add these up: 2 pts for handling the negative exponents correctly 1 pt if you express the exponent division correctly but make an error afterwards

Get rid of the negative exponents: $\frac{8^6}{8^1}$ Use the rules of exponent division: $8^{6-1} = 8^5$ You could also do this in one big step if you handle the exponent division correctly: $8^{-1-(-6)} = 8^{-1+6} = 8^5$	(there's no expectation you would multiply this out – it's only included for completeness) Worth 4 pts	
e) $3\log_4(1024)$ Notice the base of the logarithm and check whether 1024 is a power of 4. It is: $4^5 = 1024$. Re-write: $3\log_4 4^5$ Which you can immediately simplify to: $3 \cdot 5 = 15$	15 Worth 4 pts	Add these up: 2 pts for re-writing 1024 as the correct power of 4 1 pt for correctly using the rules of logs to bring down the exponent and simplify the logarithm
f) $2^{4x-1} = 16$ Recognize that 16 is a power of 2: $2^{4x-1} = 2^4$ For the equality to hold, the exponents must be equal: $4x - 1 = 4$ Solve: $4x = 5 \rightarrow x = \frac{5}{4}$	$x = \frac{5}{4}$ Worth 4 pts	1 pt for re-writing 16 as a power of 2 1 pt for recognizing the exponents must be equal 2 pts for both of these without correct calculations 3 pts for both and some correct calculations with a minor math error
g) $\sqrt{81\sqrt[3]{27}} = 3^x$ In order to get the x out of the exponent, we'll need to try to get the left side written as a power of 3. Start by simplifying and getting rid of the root symbols: $\sqrt{81\sqrt[3]{27}} = \sqrt{(9^2)(3)} = (3^2 \cdot 3^2 \cdot 3)^{\frac{1}{2}}$ $= (3^5)^{\frac{1}{2}} = 3^{\frac{5}{2}}$ Now re-write the equation: $3^{5/2} = 3^x$ So the solution is:	$x = \frac{5}{2}$ Worth 4 pts	All of the challenge here is in simplifying the left hand side. Add these up: 1 pt for recognizing $\sqrt[3]{27} = 3$ 1 pt for breaking up $81 = 3^4$ 1 pt for handling the square root appropriately 1 pt for correctly combining whatever powers of 3 you came up with (even if you made a mistake with one of the above points) 3 pts if you got to $3^{5/2} = 3^x$ but did not solve correctly.

$x = \frac{5}{2}$		
h) $e^x = 8^{3x-1}$ There are two ways to solve this. They produce expressions that look different as written, but have the same numerical value. The first is the answer we see most often. Technically, you could also use a base-2 log (since 8 is a power of 2) – it would proceed like the second approach below, except you would have to re-write 8 as 2^3 and handle that exponent appropriately – but it's more work. (1): Use the natural log – $\ln e^x = \ln 8^{3x-1}$ Bring down the exponents: $x \ln e = (3x-1)\ln 8$ Simplify the right side: $(3x-1)\ln 8 = (3x-1)\ln 2^3$ $= 3(3x-1)\ln 2$ $= 9\ln 2\,(x) - 3\ln 2$ Write the equation, noticing that $\ln e = 1$: $x = 9\ln 2\,(x) - 3\ln 2$ Solve for x: $x(1-9\ln 2) = -3\ln 2$ $x = -\frac{3\ln 2}{1-9\ln 2}$ It's not required, but you can absorb the negative sign: $x = \frac{3\ln 2}{9\ln 2 - 1}$ (2): Use the base-8 log – $\log_8 e^x = \log_8 8^{3x-1}$ Bring out the exponents: $x\log_8 e = (3x-1)\log_8 8$ Simplify, since $\log_8 8 = 1$:	Any of these (they're numerically equivalent): $x = -\frac{3\ln 2}{1-9\ln 2}$ $x = \frac{3\ln 2}{9\ln 2 - 1}$ $x = \frac{1}{3 - \log_8 e}$ $x = \frac{3}{9 - \log_2 e}$ Worth 4 pts	This is a complex question. Use your best (conservative) judgment about how far you got on the problem and award partial credit based on what share of the process you completed correctly.

$x\log_8 e = (3x-1)$ Solve: $1 = 3x - x\log_8 e$ $x(3-\log_8 e) = 1$ $x = \dfrac{1}{3-\log_8 e}$		
i) $2^{4-x^2} \le \frac{1}{8}$ Notice that the x is in the exponent, so you'll need to bring it down somehow. Check the right side to see whether it can be expressed as an exponent of 2. It can: $2^{4-x^2} \le 2^{-3}$ Simplify: $4 - x^2 \le -3$ Solve: $x^2 \ge 7$ $x \le -\sqrt{7} \cup x \ge \sqrt{7}$ Notice the solution set is expressed as the union of the sets produced by the negative a positive roots. If you're not clear on the simplification step, recognize that you could do this step: $\log_2\left(2^{4-x^2}\right) \le \log_2 2^{-3}$, and then bring out the exponent: $(4-x^2)\log_2 2 \le -3\log_2 2$, which trivially simplifies as above ($\log_2 2 = 1$).	Either $x \le -\sqrt{7} \cup x \ge \sqrt{7}$ Or $\{x\colon x \le -\sqrt{7}\ or\ x \ge \sqrt{7}\}$ (We'll accept answers that make it clear you understand that both are part of the solution) Worth 4 pts	Add these up: 1 pt for expressing $\frac{1}{8}$ as a power of 2 1 pt for simplifying correctly No partial credit beyond that point – you should only get 2 pts if you got to the solve line but only found positive portion of the solution.
j) To solve this problem, repeatedly apply rules for algebra with logarithms: $\log(ab) = \log(a) + \log(b)$ $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ $\log a^b = b\log a$ So, first split it up into four terms – one for the constants and one each for the variables $\ln\dfrac{(8\,x^3\,y^7)}{2\sqrt{z}}$	Either $\ln 4 + 3\ln x + 7\ln y - \frac{1}{2}\ln z$ or $2\ln 2 + 3\ln x + 7\ln y - \frac{1}{2}\ln z$ Worth 4 pts	1 pt each for correct use of the three rules

$= \ln\left(\frac{8}{2}\right) + \ln x^3 + \ln y^7 - \ln z^{\frac{1}{2}}$ $= \ln 4 + 3\ln x + 7\ln y - \frac{1}{2}\ln z$		

IV. Summation Notation

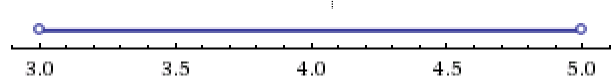
Summation notation appears frequently in Harris courses, usually in a context where you need to quickly be able to expand the expression in your mind. This is mostly a question of practice and learning a few quick simplification rules. Pay attention to the starting index (on the bottom); many mistakes come from accidentally assuming the start point is 0 or 1.

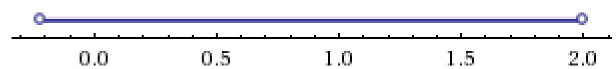
Work	Accepted answers	Grading guidance
a) $= \sum_{i=0}^{5} x^i + \sum_{i=0}^{5} y$ $= x^0 + x^1 + x^2 + x^3 + x^4 + x^5$ $+y + y + y + y + y + y$ $= 1 + x + x^2 + x^3 + x^4 + x^5 + 6y$	$1 + x + x^2 + x^3$ $+x^4 + x^5 + 6y$ The order of the terms is unimportant, as long as all are included Worth 5 pts	2 pts each for expanding either expression correctly 4 pts if you got to the second line of work but made a simplification error
b) $= \sum_{j=2}^{5} 5j + \sum_{j=2}^{5} \frac{j-1}{2} + \sum_{j=2}^{5} 2^j$	135 Worth 5 pts	1 pt each for expanding the three expressions correctly 1 pt for intermediate simplification (combining fractions in the second terms and calculating exponents in the third term) 1 pt for final addition

$= 10 + 15 + 20 + 25$ $+\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2}$ $+2^2 + 2^3 + 2^4 + 2^5$ $= 70 + 5 + 4 + 8 + 16 + 32$ $= 135$		

V. Absolute Value & Inequalities

Beyond routine math errors, the mistakes we see most frequently on these problems are (1) failing to handle multiplication or division by a negative correctly (or erroneously switching the direction of the inequality when it's not necessary, (2) failing to consider all the "cases" that might make an expression true, and (3) failure to handle the end points correctly.

Work	Accepted answers	Grading guidance
a) Factor the expression: $(x-5)(x-3) < 0$ The left hand side is a quadratic with roots at 5 and 3 – so it crosses the x-axis at those points (note that neither is a double root). Pick a test point in each of the relevant regions: $(-\infty, 3), (3,5), (5, \infty)$ Any test point you pick is fine within any of the regions – I've picked the one below for ease of computation – plug each in and check whether the assertion is true. $x = 0 \;:\; (-5)(-3) = 15 < 0 \quad FALSE$ You only need to check one point – the sign alternates for each region (as long as there are no double roots) – but it's fine if you checked each region. Since my test point from $(-\infty, 3)$ was not less than zero, I know that the expression is negative on $(3,5)$ and positive on $(5, \infty)$.	Either $(3,5)$ or $3 < x < 5$ For the graph (note the open circles on the end points indicating they are not included in the solution set – required for full credit):  Worth 5 pts for the solution + 2 pts for the	Add up: 1 pt if you showed evidence you knew you needed to find the roots, but didn't use either factoring or the quadratic formula OR 2 pts if you attempted to use one of those approaches but did so incorrectly OR 3 pts if you did so correctly 1 pt for trying test points but coming to the incorrect conclusion OR 2 pts for doing so correctly (it's ok if you're doing it for the wrong regions based on a mistake above – the points are for doing the technique correctly 2 pts for correctly graphing whatever solution you gave, but only 1 point if you did not correctly represent

	graph	the end points
b) You need to set up two cases based on the two possible values inside the absolute value sign. Case 1: positive number $6x - 2 < 3x + 4$ $3x < 6$ $x < 2$ Case 2: negative number $6x - 2 > -3x - 4$ $9x > -2$ $x > -\frac{2}{9}$ Combine the resulting two solutions for the valid range: $-\frac{2}{9} < x < 2$	Either $\left(-\frac{2}{9}, 2\right)$ or $-\frac{2}{9} < x < 2$  Worth 5 pts for the solution + 2 pts for the graph	2 pts if you set up case 1 correctly but either missed case 2 or didn't do anything close to the correct setup 3 pts if you set up cases but made a mistake in the setup (e.g. if you failed to flip the inequality for case 2) 4 pts if you did correctly but made a simple math error 2 pts for correctly graphing whatever solution you gave, but only 1 point if you did not correctly represent the end points
c) The absolute value spawns two possible solutions, one where the contents of the absolute value are positive and the other where they are negative. $8x - 1 = 2 \rightarrow \quad x = \frac{3}{8}$ $8x - 1 = -2 \ \rightarrow \quad x = -\frac{1}{8}$	$x = -\frac{1}{8}$ AND $x = \frac{3}{8}$ Worth 5 pts	2 pts for the positive solution without anything else OR 3 pts for the negative solution without anything else 2 pts for knowing you need two cases but not setting them up correctly 3 pts for setting one up correctly but a mistake in the other setup 4 pts for correct setup with minor math mistake
d) We need to consider two cases – the positive and negative values that produce the term in the absolute value. For the positive value: Case 1: $3x^2 + 36 = 24x$ $3x^2 - 24x + 36 = 0$	Four values of x solve the equation: $-6, -2, 2, 6$ Worth 5 pts	2 pts for correctly solving either case and ignoring the other 3 pts for correctly solving one and setting up the other incorrectly 4 pts for correct with minor math mistake (but must have set up both cases correctly)

$$(3x-6)(x-6)=0$$ $$x=6, x=2$$ For the negative value, Case 2: $$-(3x^2+36)=24x$$ $$-3x^2-24x-36=0$$ $$3x^2+24x+36=0$$ $$(3x+6)(x+6)=0$$ $$x=-6, x=-2$$		

VI. Functions

Handling function appropriately is foundational for everything you will do in Calculus. In particular, learning to think about functions as composites of other functions is crucial for understanding how to take derivatives correctly. Domain problems are usually a question of thinking through the values of the variable that would not result in a real-valued function. Range problems typically require you think carefully about how the function will 15behave as you allow the variable to take on all of its possible values. The final problem isn't really about functions – but about handling parameters appropriately.

<u>Work</u>	<u>Accepted answers</u>	<u>Grading guidance</u>
a) Simply plug in: $$f(x)=\sqrt{x^2+16}$$ $$f(3)=\sqrt{3^2+16}$$ $$=\sqrt{9+16}=\sqrt{25}$$ $$=5$$	$$f(3)=5$$ Worth 3 pts	1 pt if some evidence of valid setup 2 pts if plugged in correctly but had a math error
b) Plug in: $$h(z)=\frac{10}{z^2-1}$$ $$h(2)=\frac{10}{2^2-1}$$ $$=\frac{10}{3}$$	$$h(2)=\frac{10}{3}$$ Worth 3 pts	1 pt if some evidence of valid setup 2 pts if plugged in correctly but had a math error

c) The domain is the set of values of z for which the function is defined. For a function with the variable in the denominator, check for values that would make the denominator zero: $$z^2 - 1 = 0$$ $$z^2 = 1$$ $$z = \pm 1$$ In other words, those values are NOT part of the domain.	There are multiple correct ways of expressing the answer. To be acceptable, your answer must indicate that all real numbers are in the domain except -1 and 1 (a sentence like that is ok). For example: $$\mathbb{R} \notin -1, 1$$ or: $$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$ Worth 3 pts	No points unless you recognized the need to check where the denominator is zero 1 pt each for excluding -1 and 1 1 pt for including the rest of the real line
d) The range is the values that the function can take on. In this case, the square root guarantees there will only be positive values. As you look closer, notice that the x is squared, so the smallest value that can take on is 0. When that happens, the value of the function is 4. Notice that any value of x smaller or larger than zero will result in a larger value for the function. Also notice that as x increases, there is no limit on how large the value of the function can become.	Again, lots of ways to express this. Your answer should show a range from 4 to positive infinity (including 4) – a sentence is fine. You could also say $[4, \infty)$ or $\{f \in \mathbb{R}: f \geq 4\}$ Worth 3 pts	Use your judgment, based on how well you showed a line of reasoning consistent with the logic explained here. Be sure to judge based only on what is written on the paper (not what you were thinking when you wrote your answer).
e) Form the composite function by plugging $f(x)$ into $h(\quad)$: $$h \circ f(x) = \frac{10}{\left(\sqrt{x^2+16}\right)^2 - 1} = \frac{10}{x^2 + 16 - 1}$$ $$= \frac{10}{x^2+15}$$ NOTE: it's only ok to cancel the square and square root	$$h \circ f(x) = \frac{10}{x^2+15}$$ Worth 4 pts	1 pt if you showed some evidence of understanding what the composite operator means 2 pts if you substituted the wrong function 3 pts if you substituted correctly but made a simple math mistake

this way because the term inside ($x^2 + 16$) never takes on negative values. If that weren't the case, you would have to explicitly exclude from the domain the values of x that make that term negative.		
f) Substitute $y = 3$: $$g(3) = \frac{6k}{1-3k} = 2$$ Solve for k: $$6k = 2(1-3k)$$ $$6k = 2 - 6k$$ $$12k = 2$$ $$k = \frac{1}{6}$$	$k = \frac{1}{6}$ Worth 3 pts	1 pt for correct substitution of $y = 3$ 1 pt for substantial progress towards solving resulting equation for k

VII. Polynomials

The best way to study this is simply practice. Practice factoring. Practice using FOIL (first, outside, inside, last) to expand a polynomial from its factors. Practice using the quadratic formula. Learn to recognize the common, easily-factorable forms. Make sure your practice includes polynomials with mixture of integer, fraction, and parameter coefficients.

Work	Accepted answers	Grading guidance
a) $y = (x-3)(x+8)$	$y = (x-3)(x+8)$ (you can have the two terms switched, which is equivalent) Worth 3 pts	There's really only one way to do this correctly. Feel free to give a point if you showed signs of getting toward the right solution.
b) Solve $0 = (x-3)(x+8)$ for the x-intercepts. $$x - 3 = 0 \rightarrow x = 3$$	x-intercepts: 3 & -8 y-intercept: -24	Add together: 1 pt for correct setup for both x-intercept calculations

$x + 8 = 0 \rightarrow x = -8$ Solve $y = (0 - 3)(0 + 8) = -24$ for the y-intercept.	Worth 5 pts	1 pt each for the x-intercepts 1 pt for setting up the calculation of the y-intercept correctly 1 pt for the y-intercept If you did the factoring in part a) correctly, give yourself full credit on the 3 pts for the x-intercepts if you used the right process to determine the x-intercepts based on that answer
c) Plug in the y value: $-18 = x^2 + 5x - 24$ Solve: $x^2 + 5x - 6 = 0$ Factor: $(x + 6)(x - 1) = 0$ Either 6 or 1 solve this equation, so the graph of the original polynomial passes through $(-6, -18)$ and $(1, -18)$	$x = -6$ $x = 1$ Worth 4 pts	1 pt for plugging in the -18 without additional effort 2 pts for solving and factoring correctly without providing evidence you knew how to translate that into a solution 3 pts for solving and factoring incorrectly and correctly interpreting the result
d) This doesn't factor easily, so use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{7 \pm \sqrt{49 - 4(3)(2)}}{2(3)}$ $x = \frac{7 \pm \sqrt{49 - 24}}{6} = \frac{7 \pm \sqrt{25}}{6}$ $x = \frac{7 \pm 5}{6}$ $x = 2, x = \frac{1}{3}$	$x = 2$ $x = \frac{1}{3}$ Worth 4 pts	1 pt if you tried to factor unsuccessfully 2 pts for the quadratic formula 3 pts if you correctly plugged into the formula but made a math error calculating the root

e) Examine the expression for common factors, and notice you can factor out an a: $$a^3 - 9ab^2 = a(a^2 - 9b^2)$$ Now consider the second term to determine whether it can be factored. Notice all the terms are squares and it can easily be factored farther: $$a(a^2 - 9b^2) = a(a - 3b)(a + 3b)$$ So, $$y = a(a - 3b)(a + 3b)$$	$y = a(a - 3b)(a + 3b)$ The order of the factors is unimportant – these 3 can be in any order and be correct.	1 pt for factoring out the a 2 pts if you factored the a and made a serious attempt to factor further (perhaps with a math error)