

## Algebra Example Exam

We strongly encourage students to sit this exam as if it were the real thing – take it in a quiet setting all at one time, avoid using notes or calculators, and enforce the time limit. Use the scoring guide to grade your own paper and check whether you would pass the exam.

### Instructions:

- This exam is closed book, closed notes, no calculators. You may use a ruler if you like.
- You have 1½ hours to complete the exam.
- Show your work in the space provided – there is partial credit given on the real test!
- Put your answers in the boxes or spaces provided for each problem.
- Your answers should be simplified – e.g. take square roots with integer solutions, reduce fractions, etc. If a root or logarithm has a non-integer value, leave it in the precise form

**SCORE:** \_\_\_\_\_ / 150 points

The exam has the following seven sections. Scores of 60% or better will pass the exam. Please budget your time accordingly.

Linear equations:	16 pts
Systems of equations & graphing:	21 pts
Exponents & logarithms:	40 pts
Summation notation:	10 pts
Absolute values & inequalities:	24 pts
Functions:	19 pts
Polynomials:	20 pts

## I. Linear Equations

Consider the points  $A = (8,1)$ ,  $B = (-4,2)$

- a) (3 pts) What is the slope of the line between the points?

Answer:

- b) (4 pts) What is the equation for the line containing both points?

Answer:

- c) (3 pts) What is the distance between points A & B?

Answer:

- d) (3 pts) What is the slope of any line perpendicular to the line from (b) above?

Answer:

- e) (3 pts) What is the equation for a line parallel to the line from (b) and passing through the point  $(5,0)$ ?

Answer:

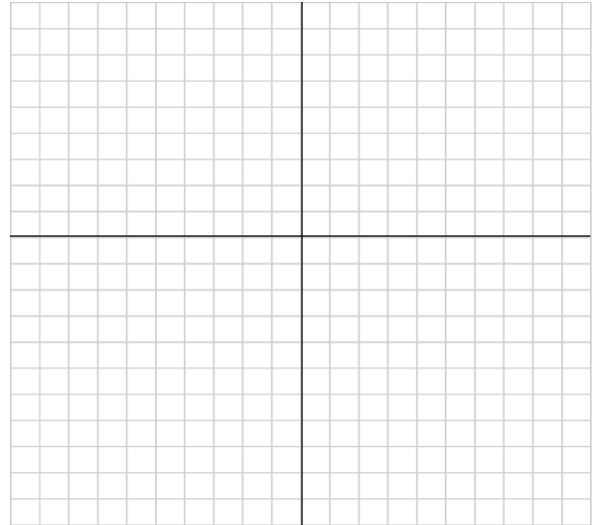
## II. Systems of Equations

- a) (4 pts) Graph the system:

$$\begin{cases} y = \frac{2}{3}x + 2 \\ y = -\frac{1}{3}x - 1 \end{cases}$$

- b) (3 pts) Solve the system above.

Answer:



- c) (4 pts) Find the precise area of the triangle formed by the two lines and the y-axis.

Answer:

- d) (4 pts) Solve the system:  $\begin{cases} 8x + 2y = 8 \\ -x + y = -6 \end{cases}$

Answer:

- e) (6 pts) Solve the system:  $\begin{cases} x + y + z = 6 \\ 3x - 2y + 4z = 9 \\ x - y - z = 0 \end{cases}$

Answer:

### III. Exponents & Logarithms

For this section, you do not need to expand out exponents (e.g., leave  $3^4$  rather than writing 81). If it is possible to evaluate a logarithm easily, you should do so; however if there is no simple integer or fraction value for the logarithm, it is fine for your solution to include a logarithm (e.g. you should simplify things like  $\ln e^4$  but leave  $\log_5 2$  as is).

Simplify these expressions as much as possible (4 pts each).

a)  $\left(\frac{36}{81}\right)^{-1/2}$

Answer:

b)  $\frac{(2x^{-2}y^3)}{(8x^5y^{-8})}$

Answer:

c)  $\log_5(0.2)$

Answer:

d)  $\frac{8^{-1}}{8^{-6}}$

Answer:

e)  $3 \log_4 1024$

Answer:

Solve for x (4 pts each):

f)  $2^{4x-1} = 16$

Answer:

g)  $\sqrt{81\sqrt[3]{27}} = 3^x$

Answer:

h)  $e^x = 8^{3x-1}$

Answer:

i) Solve the inequality  $2^{4-x^2} \leq \frac{1}{8}$

Answer:

- j) Use the rules of logs to rewrite this expression without multiplication, division, or exponents (expand it out – you may still have some expressions of the form:  $a \ln b$ ):

$$\ln \frac{(8x^3y^7)}{2\sqrt{z}}$$

Answer:

#### IV. Summation Notation

- a) (5 pts) Expand the expression below (re-write it without using summation notation). Simplify completely.

$$\sum_{i=0}^5 (x^i + y) =$$

Answer:

- b) (5 pts) Calculate the value of the following:

$$\sum_{j=2}^5 5j + \frac{j-1}{2} + 2^j =$$

Answer:

## V. Absolute Value & Inequalities

Find all values of  $x$  for which these expressions hold true. Graph the solution sets on the number line, clearly indicating the inclusion/exclusion of any end points.

a) (7 pts)

$$x^2 - 8x + 15 < 0$$



Answer(s):

b) (7 pts)

$$|6x - 2| < 3x + 4$$



Answer(s):

Find all solutions to the equations below.

c) (5 pts)

$$|8x - 1| = 2$$

Answer(s):

d) (5 pts)

$$3x^2 + 36 = |24x|$$

Answer(s):

VI. Functions

$$f(x) = \sqrt{x^2 + 16} \quad h(z) = \frac{10}{z^2 - 1} \quad g(y) = \frac{2ky}{1 - ky}$$

a) (3 pts) Evaluate  $f(3)$

Answer:

b) (3 pts) Evaluate  $h(2)$

Answer:

c) (3 pts) What is the domain of  $h(z)$ ? \_\_\_\_\_

d) (3 pts) What is the range of  $f(x)$ ? \_\_\_\_\_

e) (4 pts) Find  $h \circ f(x)$

\_\_\_\_\_

f) (3 pts)  $g(y)$  is defined in terms of parameter  $k$ . What value of the parameter would produce  $(3) = 2$  ?

Answer:



VII. Polynomials

- a) (3 pts) Factor the polynomial  $y = x^2 + 5x - 24$

Answer(s):

- b) (5 pts) What are the  $x$ - and  $y$ -intercept(s) when this polynomial is graphed? (0, 1, or 2 possible)

$x$ -intercept(s):

$y$ -intercept(s):

- c) (4 pts) For what value(s) of  $x$  does the graph of this function pass through  $y = -18$ ?

Answer(s):

- d) (4 pts) Find the roots of the polynomial  $y = 3x^2 - 7x + 2$

Answer(s):

- e) (4 pts) Factor the polynomial  $y = a^3 - 9ab^2$  as completely as possible.

Answer(s):