Math Self-Assessment

Instructions

- You have 1.5 hours to complete the exam. Calculators are not allowed.
- Show your work in the space provided – there is partial credit! In particular, please write out formulas before you use them.
- Your answers should be simplified – e.g. take square roots with integer solutions, reduce fractions, etc. If a root or logarithm has a non-integer value, leave it in the precise form – e.g. do not attempt to write the approximate decimal values for ln 2 or 3.

Question 1 (5 points)

Solve for x. Be sure to check your work:

$$5x^2 + 10x = -5$$

Question 2 (5 points)

Solve for x. Be sure to check your work:

$$\frac{1}{2}\sqrt{4x + 8} = x$$

Question 3 (10 points)

$$Q = \frac{2}{p_1 p_2} + \frac{4p_1 - 5}{p_1 + p_2}$$

(a)   Find the derivative of Q with respect to p_1

(b)   Find the derivative of Q with respect to p_2

Question 4 (5 points)

Suppose x_1 is the number of hours you study for the GRE, x_2 is your parental income (measured in tens of thousands of dollars), and y is your GRE score in the following equation:

$$y = 45 + 2.2x_1 + 4x_2$$

(a)   Interpret the coefficients of the equation.

(b)   How much does one's expected GRE score increase by when they study 5 more hours? What about when their parental income increases by $5,000?

Question 5 (5 points)

True or False (with a brief 1-2 sentence explanation)

The median is more affected by outliers than the mean.

Question 6 (10 points)

Suppose $U = x^2 y^3$.

    (a)   Solve for the ratio of the first derivatives:

$$\frac{\dfrac{\partial U}{\partial x}}{\dfrac{\partial U}{\partial y}}$$

    (b)   This ratio is called the "Marginal Rate of Substitution" (MRS). Does it increase or decrease as x increases?

Question 7 (10 points)

Suppose $Q_D = -2p + 100$ represents the demand of a good, where p is the price of the good. Graph this equation with **p on the y-axis (vertical)** and Q_D on the x-axis (horizontal). No need to make the graph extremely pointwise precise, but just make the intercept and slope clearly visible.

If $Q_S = 2p + 40$ represents the quantity supplied, find the price where $Q_D = Q_S$ is true. This point of intersection is called the "Equilibrium". What is the equilibrium price **and** quantity?

Question 8 (10 points)

Suppose you have $3000 available to spend on Macs and cheese. Macs cost $500 each, and cheese costs $100 each (hey, it's high quality!). Write down an expression that represents your total expenditure on Macs and Cheese.

If you spent the entire $3,000, write down an equation that gives you the amount of cheese you can purchase if you purchase x units of Macs. Be sure to clearly define the variables you use. (Note: The slope of the equation above is called the "price ratio" and should equal the price of good x divided by the price of good y.)

Question 9 (10 points)

Calculate Mean, Median, and Sample Variance of the following dataset:

$$\{6, 6, 12, 9, 7\}$$

Hint: The formula for Sample Variance is

$$\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

where $\bar{x}$ is the sample mean and N is the sample size

Question 10 (10 points)

Political scientists often are tasked with describing someone's political ideology using a single number. Suppose professor McMillon at the University of Chicago has developed a scale ranging from -5 to 5, where -5 means extremely progressive and 5 means extremely conservative, and 0 is an exact moderate. Sarah Palin is considering entering the presidential race.

    (a)   We know that Donald Trump's ideology is a 4.5, and that Sarah Palin's ideology, S, is within 6 units of Trump's. Represent Sarah Palin's ideology using an **absolute value inequality**, and then solve this inequality to get a range for S.

    (b)   We know that Joe Biden's ideology is a -2, and that Sarah Palin's ideology is within 2 units of Biden's. Represent Sarah Palin's ideology using an **absolute value inequality**, and then solve this inequality to get a range for S.

    (c)   If both (a) and (b) above are true, give an overall range of possible ideologies for Sarah Palin. (Note: If you think no such range exists, explain why).

Question 11 (10 points)

Suppose $U = 2xy$. Subject to the constraint that $x + 2y = 40$, solve for the x and y values that maximize U.

Question 12 (10 points)

The price of Charmin Ultra-soft Toilet Paper is given by the expression $P = 100 - y$, where y is the number of rolls sold. If the cost of producing y rolls of toilet paper is given by $C = y^2$, then what quantity y and price P would maximize the profits from selling toilet paper? Hint: The formula for profit is Revenue minus Cost, where Revenue is price (P) times quantity sold (y).